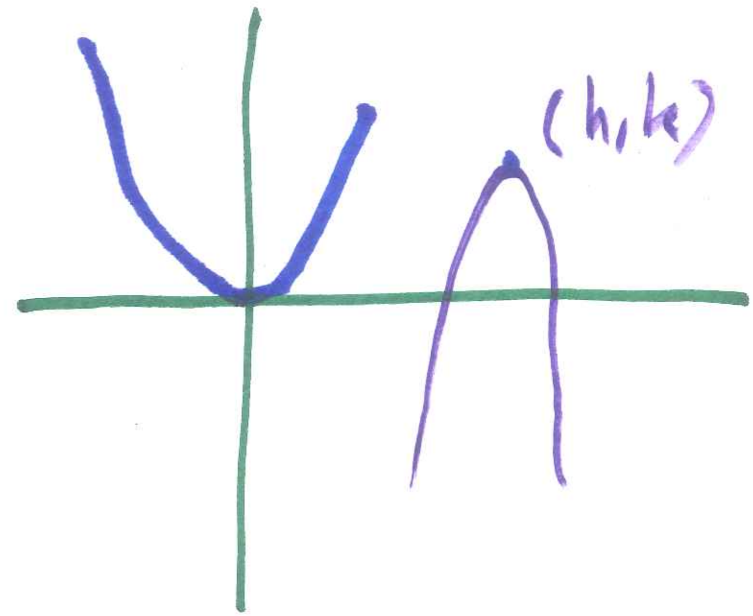
MA110. 9/26.

- Office hours -
Mathskeller. today 10-11
- Quiz Thursday.

The function

$f(x) = a(x-h)^2 + k$

has a graph obtained by rescaling $g(x) = x^2$, reflecting in the x-axis if $a<0$, translating h units to right and k units up



The point (h,k) is called the vertex of the quadratic function f.

Which number x is the "best" approximation to 2 and 10.

minimize residuals.

Precise problem: Minimize the sum of the squares of the residuals.

$$R(x) = (x-2)^2 + (x-10)^2$$

The x-coordinate of the vertex will

$R(x) =$

$$x^2 - 4x + 4 + x^2 - 20x + 100$$

$$= 2x^2 - 24x + 104$$

$$= 2(x^2 - 12x + 36 - 36) + 104$$

$$= 2(x-6)^2 - 72 + 104$$

$$= 2(x-6)^2 + 32.$$

6 is the "best" approx.

What happens if we have 3 numbers?

Polynomials.

If $a_0, a_1, a_2 \ldots, a_n$ are numbers, $a_n \neq 0$

Then

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots$$
$$\ldots + a_1 x + a_0.$$

We call the highest power n, the degree of P.

- All the exponents $0, 1, 2 \ldots, n$ are non-negative integers.

$t^3 + 3t^2$ is a polynomial.

$2x^3 + 3x^{-2} + 42$ is not a polynomial since one exponent is negative.

$P(x) = 10 = 10x^0$

Domain is always all real numbers for a polynomial.

Arithmetic with polynomials.

- Addition
- Subtraction
- Multiplication.

Simplify

$(x^2+1)(x^3-2x-4)$.

$$= x^2(x^3-2x-4)$$

$$+ 1\cdot(x^3-2x-4).$$

$$= x^5 - 2x^3 - 4x^2$$

$$+x^3 \qquad -2x - 4$$

$$= x^5 - x^3 - 4x^2 - 2x - 4.$$

Recall division of integers -

Divide 3 into 340.

$$\begin{array}{r} 113 \\ 3\overline{)\,340} \\ \underline{300} \\ 40 \\ \underline{30} \\ 10 \\ \underline{9} \\ 1 \end{array}$$

113 is the quotient.

1 is the remainder

3 is the divisor

340 is the dividend.

$$340 = 3 \cdot 113 + 1.$$

Require remainder to be less than the divisor.

Note

$$340 = 3 \cdot 10^2 + 4 \cdot 10^1 + 0 \cdot 10^0$$

If P & D are polynomials

we can write

$$P = D \cdot Q + R$$

where Q, R are

polynomials &

degree of R is

less than the degree

of D.

Divide $x+2$ into x^3+1.

$$\begin{array}{r|l} & x^2 - 2x + 4 \\ \hline x+2 & x^3 + 0x^2 + 0x + 1 \\ & x^3 + 2x^2 \\ \hline & 0 \ -2x^2 + 0x + 1 \\ & -2x^2 - 4x \\ \hline & 0 \ +4x + 1. \\ & 4x + 8 \\ \hline & 0 \ -7. \end{array}$$

Check

$$x^3+1 \stackrel{?}{=} (x+2)(x^2-2x+4)$$

$$-7.$$

Roots of Polynomials.

or solutions of

$P(x)=0$

How many?

Suppose $P(c)=0$

Divide P by $x-c$. Write

$P(x) = (x-c)Q(x) + R.$

where R, the remainder is a number.

Sub. $x=c$

$P(c) = \underbrace{(c-c)}_{=0}Q(c) + R$

$P(c) = R = 0$

$P(x)$ is of degree n.

$P(c_n) = 0$

$P(x) = Q(x)(x-c_n).$

with Q of degree $n-1$.

If $P(c_{n-1}) = 0$.

$P(x) = (x-c_n)(x-c_{n-1})\cdot\tilde{Q}(x).$

$\tilde{Q}$ is of degree $n-2$.

P can have at most n roots.