MA110. Lecture 16.
Friday 9/30/16.

- Today's office hours ~~10-11~~, not 11-12, not ~~9-10~~, 10-11.

Rational functions.

A rational function is a function $R(x)$ of the form

$$R(x) = \frac{P(x)}{Q(x)}$$

where P, Q are polynomials.

Example. Every polynomial

$$P(x) = \frac{P(x)}{1}.$$

$$Q(x) = \frac{1}{x^2+1}$$

$$R(x) = \frac{1}{x}$$

$$T(x) = \frac{1}{x-1} + \frac{1}{x+1}.$$

$= \frac{1}{(x-1)(x+1)}\frac{(x+1)}{} + \frac{1}{x+1}\frac{(x-1)}{(x-1)}$

$= \frac{(x+1) + (x-1)}{(x-1)(x+1)}.$

$= \frac{2x}{(x^2-1)}.$

Reel. #1

$\frac{1}{x^2-x-20} = \frac{1}{(x-5)(x+4)}$

Check.

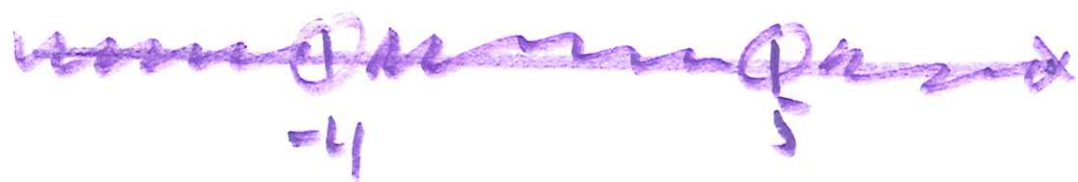
$(x-5)(x+4)$

$= x(x+4) - 5(x+4)$

$= x^2 + \underbrace{4x - 5x} - 20$

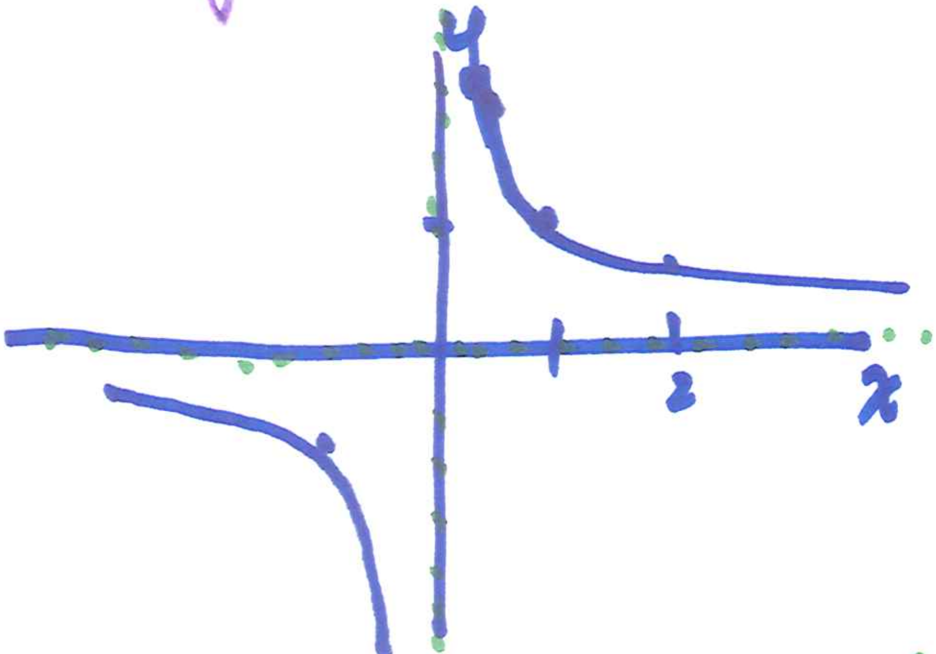
$= x^2 - x - 20$ ✓

Domain is x such that $x \neq -4$ and $x \neq 5$.



$(-\infty, -4) \cup (-4, 5) \cup (5, \infty).$

Graph $f(x) = \frac{1}{x}$



x	y
$1/2$	2
1	1
2	$1/2$
$-1/2$	-2
-1	-1
-2	$-1/2$

Note:

- If x is small, $f(x)$ is large &
- If x is large $1/x$ is small.
- If ~~as~~ the graph of a function approaches a line, as we move away from $(0,0)$, then the line is called an asymptote of the function.

Vertical asymptotes are lines $x = c$

Horizontal asymptotes are ones of the form $y = c$.

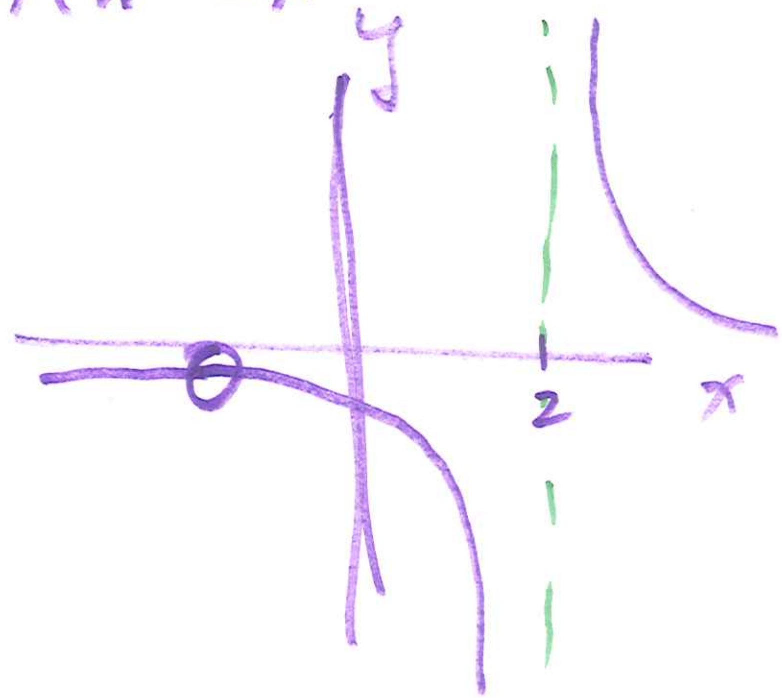
Example $y = \frac{1}{x}$ has asymptotes $y=0$, and $x=0$

Reel #2.

Find asymptotes of $g(x) = \frac{x+2}{x^2-4}$.

$= \frac{\cancel{x+2}}{(\cancel{x+2})(x-2)}$ if $x \neq -2$.

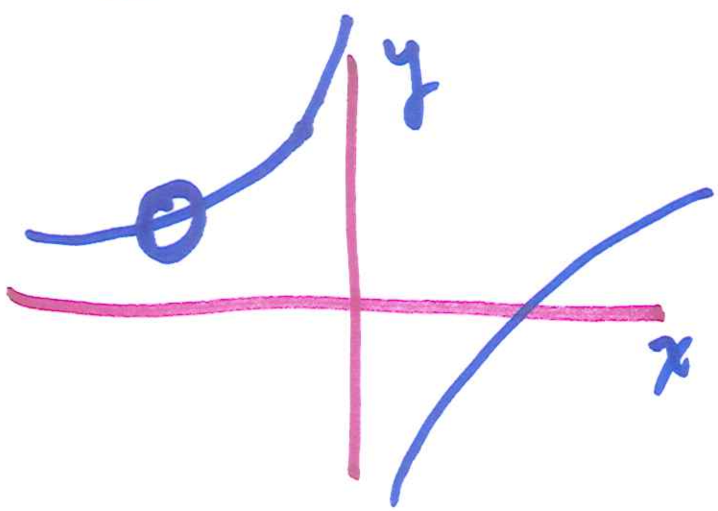
$\frac{1}{x-2}$



If a rational function simplifies to give

$$R(x) = \frac{(x-c)}{(x-c)} \frac{P(x)}{Q(x)}$$

and $Q(c) \neq 0$, then the graph of R has a hole at c.



Rational linear functions –

$$P(x) = \frac{ax+b}{cx+d}.$$

Assume $c \neq 0$ and $ad \neq bc$

Find asymptotes –

Vertical asymptote if $cx+d=0$ and $ax+b \neq 0$

That $x = -\frac{d}{c}$

and $-\frac{d}{c} \neq -b/a$

or $ad \neq bc$.

Horiz. asymptote.

$$P(x) = \frac{x}{x}\left(\frac{a + b/x}{c + d/x}\right)$$

As x gets large, ~~Bo~~ b/x and d/x become small and the graph of P approaches the line $y = a/c$.

Since $c \neq 0$, $y = a/c$ is a horiz. asymptote.

REEF # 3.

Find A, B so that

$$f(x) = \frac{6x+7}{Ax+B}$$

has a horizontal asymptote at $y=2$ and vertical asympt. at $x=-3$.

Horiz. asympt

if $y = 6/A = 2$

or $A = 3$.

~~Vert. Asymptote~~

if

$\cancel{6x+7 = 0}$

$\cancel{Ax+B = 3x+B}$

$\cancel{= 0}$

~~or A(~~

Vertical asymptote at $x = -3, 6$

$A(-3) + B = 0$

or $-9 + B = 0$

or $\underline{B = 9}$.

Higher degree rational functions

$R(x) = \dfrac{ax^n + \dots}{bx^m + \dots}$

where $+ \dots$ stands for terms of lower degree.

$n > m$ R approaches $\frac{a}{b}x^{n-m}$, so no horiz. asymptote

$n < m$, $R(x)$ approaches 0 as x becomes large

$n = m$, $y = \frac{a}{b}$ is a horiz. asymptote.

REEF #4.

(A) $\frac{x^3+2}{x-2}$.

$3 > 1$, No asymptote

(B) $\frac{3x^3+2}{4x^4-2}$

$y=0$ is asymptote

(C) $\frac{x^3+25}{26+x^3}$.

$y=1$ ✓

(D) ~~x^4+~~

(D) $\dfrac{x^4+2x}{2x^4+x}$

has a horiz. asympt.

at $y = 1/2$.