

MA 110 - 10/3/16.

Rational & Polynomial
inequalities.

Office hours today
in Mathskeller.
10-11.

REEF #1.

If $x > y$, then.

$$-(x+y) \quad -(x+y)$$

$$\cancel{-x} - \cancel{y} + \cancel{x} > \cancel{y} - \cancel{x} - \cancel{y}$$
$$-y > -x$$

-Given a ~~an~~ true
inequality

$$a > b,$$

we can (1) add
a quantity to both
sides (2) multiply by
a positive quantity
(3) multiplying by
a negative quantity
changes the direction
of the inequality.

$$x > y$$
$$-1 \cdot x < -1 \cdot y$$

$$-x < -y$$
$$-y > -x.$$

Is $x > y$ is $x^2 > y^2$?

Try $x=1, y=-2$

$$1 > -2$$

but $1^2 > (-2)^2$ fails.

Exercise show that
 $-x > -y$ and $|x| > |y|$
may fail, also.

Division is the
same as multiplying
by the reciprocal.

Subtraction is
handled by adding
the negative

Be careful --

Don't multiply
inequalities by
expressions involving
a variable.

Solve

$$-2x + 1 < -3(2 + x).$$

$$\begin{array}{r} -2x + 1 < -6 - 3x \\ +2x \qquad +2x \\ \hline 1 < -6 - 3x + 2x \end{array}$$

$$1 < -6 - 3x + 2x.$$

$$1 < -6 - x.$$

$$+6 \qquad +6$$

$$7 < -x.$$

$$(-1) \cdot \qquad (-1) \cdot$$

$$-7 > x.$$

Solution —

$$(-\infty, -7).$$

Check.
Is 0 not a solution?
Is -8 a solution?

REEF #2.

Solve

$$2(x+2) > 3(4-2x)$$

$$\begin{array}{r} 2x + 4 > 12 - 6x \\ +6x \qquad +6x \end{array}$$

$$\begin{array}{r} 8x + 4 > 12 \\ -4 \qquad -4 \end{array}$$

$$8x > 8$$

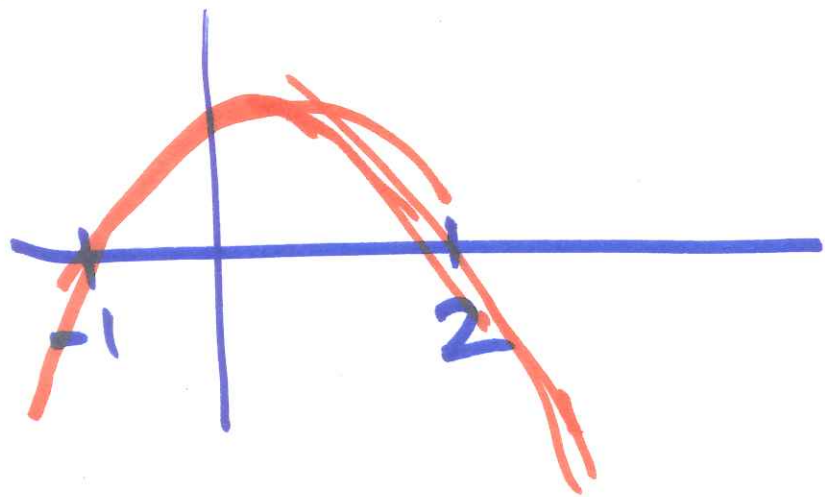
$$x > 1. \cup (1, \infty)$$

Quadratic inequality

$$\text{Ex } P(x) = -2(x+1)(x-2).$$

Solve $P(x) < 0$.

Graph of $P(x)$ is a parabola. Opens down since leading term is $-2x^2$.

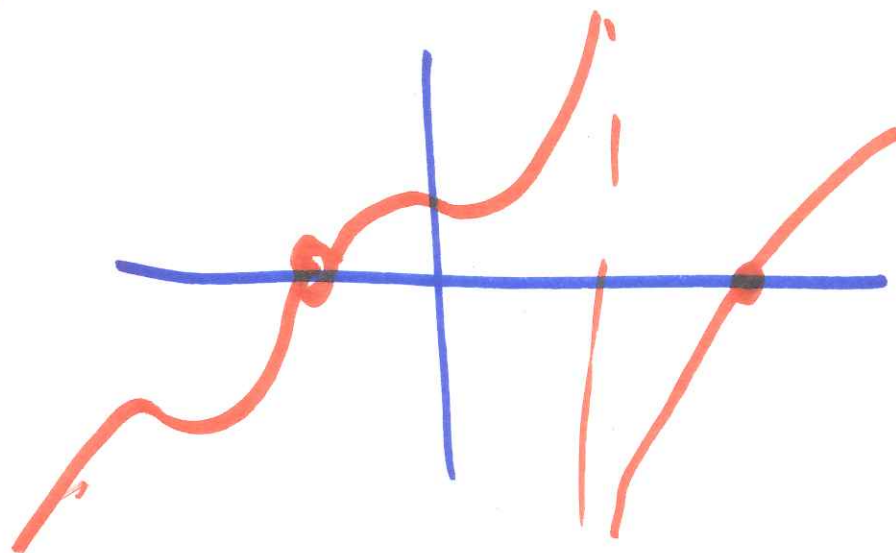


Solution is

$$(-\infty, -1) \cup (2, \infty).$$

Rational & polynomial inequalities.

—Graph may cross the x-axis at a root or a point not in the domain.



To solve

$$R(x) > 0$$

- Compare to zero.
- Find points where the function may cross the x -axis.
- Test each interval.

Solve

$$\frac{x+6}{x+2} > x.$$

$-x$

$-x$

$$R(x) = \frac{-1(x+3)(x-2)}{x+2} > 0$$

$$\frac{x+6}{x+2} - x \frac{(x+2)}{x+2} > 0.$$

$$\frac{x+6 - x^2 - 2x}{x+2} > 0.$$

$$\frac{-x^2 - x + 6}{x+2} > 0$$

$$\frac{-1(x^2 + x - 6)}{x+2} > 0$$

$$\frac{-1(x+3)(x-2)}{(x+2)} > 0$$

for $x \in$
 $(-\infty, -3) \cup (-2, 2)$

$$\frac{-1(x+3)(x-2)}{(x+2)} \text{ may cross}$$

the x -axis at $-3, -2$

at 2.

-3	-2	2
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x	-4	-5/2	0	3.
$\frac{-(x+3)(x-2)}{x+2}$	3	-9/2	3	-6/5
Sign	+	-	+	-

$$\frac{-(x+3)(x-2)}{x+2} > 0$$

REF #3.

Solve

$$\frac{1}{x+1} < \frac{1}{x+2}$$

$$0 < \frac{1}{x+2} - \frac{1}{x+1}$$

$$< \frac{(x+1) - (x+2)}{(x+1)(x+2)}$$

$$< \frac{-1}{(x+1)(x+2)}$$

$\frac{-1}{(x+1)(x+2)}$ may change
signs at $x = -1$ or -2 .

	-2	-1
x	-3	$-\frac{3}{2}$
$-\frac{1}{(x+1)(x+2)}$	$-\frac{1}{2}$	$+\frac{1}{2}$
	-	+

$$\frac{-1}{(x+1)(x+2)} > 0 \quad \text{or}$$

$$x \text{ in } \underline{(-2, -1)}.$$