MA110 - 10/5/16.

- Complex numbers.
  - What are they?
  - Rules for arithmetic
  - Solving quadratic equations

REF #1.

Solve $x^2+1=0$

or $x^2=-1$.

$i=\sqrt{-1}$, then $i^2=-1$

or $i^2+1=0$

The complex numbers are the expressions of the form

$a+bi$

where a, b are real and $i=\sqrt{-1}$. with $i^2=-1$.

Arithmetic.

$i^4 = i^2 \cdot i^2 = -1 \cdot (-1) = +1.$

$i^3 = (+i)^2 \cdot i = -1 \cdot i = -i.$

$\frac{1}{i} = \cdot i^{-1}$. Let

$x = i^{-1}$. Want $x \cdot i = 1$.

Try $x = -i$

$-i \cdot i = -(i)^2$

$= -1 = +1.$

Addition –

$(a+bi)+(c+di)$

$= a+c+bi+di$

$= a+c+(b+d)i$

For a complex number $a+bi$, a is the real part & b is the imaginary part.

Multiplication.

$(2+3i)(4-5i)$

$= 2(4-5i)+3i(4-5i)$

$= 2\cdot 4 - 2\cdot 5i + 3\cdot 4i - 3\cdot 5i^2$

$= 8 - 10i + 12i + 15$ ~~i^2~~

$= 8+15+(12-10)i$

$= 23 + 2i$

$(a+bi)(c+di)$

$= ac - bd + (ad+bc)i$

REEF #2

$(-2+3i)(2+3i)$

$= -2\cdot 2 + -2\cdot 3i$

$+3i\cdot 2 + (3i)^2$

$= -4 - 6i + 6i - 9$

$= -13 + 0i$

$= -13.$

Conjugate of $a+bi$ is $a-bi$.

Note that

$(a+bi)(a-bi)$

$= a^2 - (bi)^2$

$= a^2 - b^2 i^2 = a^2 + b^2.$

Division?

$$\frac{1}{1+2i}\ \frac{1-2i}{1-2i} = \frac{1-2i}{1-4i^2}$$

$$= \frac{1-2i}{5}$$

$$= \frac{1}{5} - \frac{2}{5}i$$

REEF #3.

$$\frac{2+3i}{1+2i}\quad\frac{(1-2i)}{(1-2i)}$$

$$= \frac{(2+3i)(1-2i)}{5}.$$

$$= \frac{1}{5}(2-6i^2+3i-4i)$$

$$= \frac{1}{5}(2+6-i)$$

$$= \frac{8}{5} - \frac{i}{5}.$$

Solving quadratic equations.

Simple case Solve

$$x^2 = -b \quad \text{with } b \geq 0$$

$$x = \pm\sqrt{-b}$$

$$= \pm\sqrt{-1 \cdot b}$$

$$= \pm\sqrt{(i)^2(\sqrt{b})^2}$$

$$= \pm i\sqrt{b}.$$

$\sqrt{-b} = i\sqrt{b}$ with $b \geq 0$.

Solve

$x^2 + x + 1 = 0$

$x^2 + x + \frac{1}{4} = -1 + \frac{1}{4}$

$(x + \frac{1}{2})^2 = -3/4$

$x + \frac{1}{2} = \pm\sqrt{-3/4}$

$= \pm i\sqrt{3/4}$ ∴

$= \pm \frac{i\sqrt{3}}{2}$ ∴

$x = -\frac{1}{2} \pm \frac{i\sqrt{3}}{2}$.

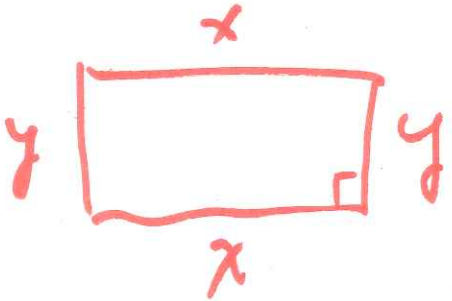
$$\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}}$$

$$= \frac{\sqrt{3}}{2}.$$

$$\sqrt{\frac{3}{5}} = \frac{\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$$

$$= \frac{\sqrt{15}}{5}.$$

The area of a rectangle is fixed at 10. Find the rectangle where the perimeter is at most 14.



$xy = 10$ or $y = \frac{10}{x}$

Perimeter is

$2x + 2y$

or

$2x + \frac{20}{x}$.

Want

$2x + \frac{20}{x} \leq 14$.

~~x^2~~

$2x + \frac{20}{x} - 14 \leq 0$.

$\frac{2x^2 + 20 - 14x}{x} \leq 0$.

Know ~~$x \neq 0$~~ $x > 0$

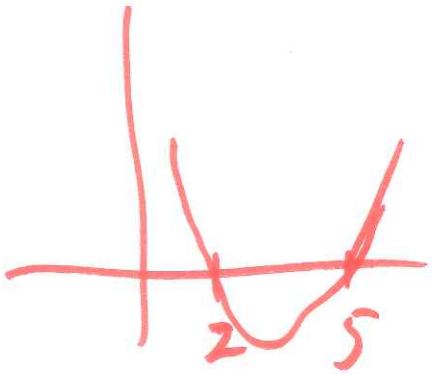
Solve

$2x^2 + 20 - 14x \leq 0$

$2(x^2 - 7x + 10) \leq 0$

$2(x-2)(x-5) \leq 0$

True if $2 \leq x \leq 5$.



The rectangle has dimensions

$x, \frac{10}{x}$,

for $2 \leq x \leq 5$.

REEF #4.

Solve

$x^2 + 4x = -5$

$x^2 + 4x + 4 = -5 + 4$

$(x+2)^2 = -1$

$x + 2 = \pm i$

$x = -2 \pm i$.