MA110. 10/10/16.

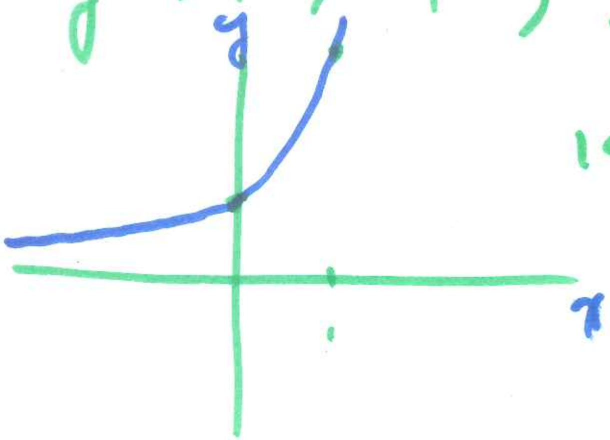
- Exam 2 on 10/18.
- Midterm grades - after exam.

Exponential functions.

$f(x) = a^x$, $a > 0$, $a \neq 1$.

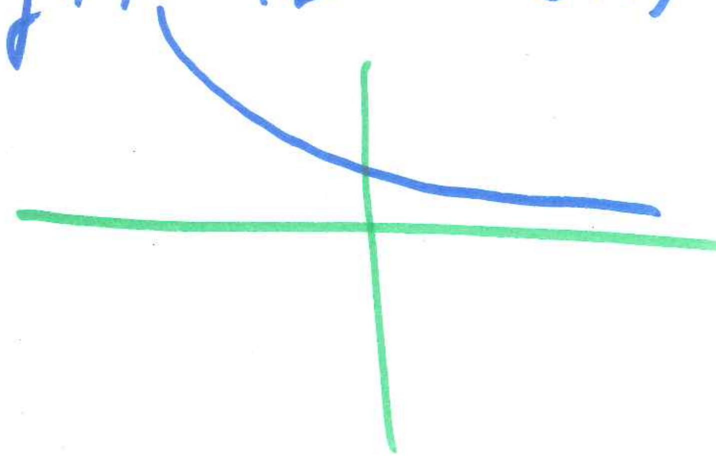
$\cancel{g(x)}\ f(x) = 42^x$, $f(0) = 1$.

y-intercept is 1.



$42 > 1$

$g(x) = 42^{-x} = \left(\frac{1}{42}\right)^x$



Example. "Exponential growth"

A substance doubles every hour. If we start w/ 10 grams how much do we have after x hours?

$x=0$	10
$x=1$	20
$x=2$	$2^2\cdot 10$
$x=3$	$2^3\cdot 10$
x	$10\cdot 2^x$.

Example: Exponential decay. $f(x) = C\cdot a^x$
$0<a<1$. $\quad = C\cdot\left(\frac{1}{a}\right)^{-x}$

Carbon 13 decays with a half-life of 5700 years. Suppose we start w/ 10 gms of Carbon 13.

How much do we have after t years?

Solution. Let $M(t)$ be ~~M(t)~~ the mass at time t.

$$M(0)=10$$

$$M(5700) = \frac{1}{2}\cdot 10 = 5.$$

$$M(11{,}400) = 2.5.$$

$$M(t) = 10\cdot 2^{-t/5700}$$

$M(t+5700)$

$= 10 \cdot 2^{-\frac{(t+5700)}{5700}}$

$= 10 \cdot 2^{-\frac{t}{5700}} \; \overset{1/2}{\cancel{2^{-\frac{5700}{5700}}}}$

$= \frac{1}{2} M(t).$

REEF #2.

Start w/ 1000.

Double every 7 yrs.

How many after 8yrs?

$P(t) = 1000 \cdot 2^{t/7}.$

$P(8) = 1000 \cdot 2^{8/7}.$

$= 2208.179\ldots$

$= \underline{2208}.$

~~P~~

Want $P(7) = 2 \cdot P(0)$

$= 2 \cdot 1000$

$= 2^{7/7} \cdot 1000$

$P(t) = 2^{t/7} \cdot 1000$

~~9th~~.

~~4~~

Pop after 8 years might be

$$1000 \cdot 2^{1/7}$$

Population might be

$$2000 \cdot 2^{1/7}.$$

$$= 1000 \cdot 2 \cdot 2^{1/7}$$

$$= 1000 \cdot 2^{1+\frac{1}{7}}$$

$$= 1000 \cdot 2^{8/7}.$$

Compound interest.

<u>Example</u>. Suppose we deposit \$101. and earn 3% per year, compounded quarterly. How much do we have after 18 months?

	Balance
0 months	101
3	$101\left(1.+\frac{0.03}{4}\right)^1$
6	$101\,(1+0.0075)^2$
⋮	

After n compoundings or $3 \cdot n$ months, we have

$$101 \cdot (1 + 0.0075)^{n}.$$

After 18 months we have

$$101 \cdot (1.0075)^6$$

$$\approx 105.63$$

Suppose we deposit 185 at 6% annual interest, compounded monthly. How much will we have after 18 months?

$$185 \cdot (1 + 0.005)^{18}$$

$$\approx 202.38.$$

Suppose we have 100% interest compounded n times. What happens when n becomes large?

n	$(1+\frac{1}{n})^n$
1	2
42	2.6866...
110	2.706...
2016	2.7176
1,000,000	2.71828....

As n approaches we obtain $(1+\frac{1}{n})^n$ approaches $e \approx 2.71828...$

If we deposit P dollars at an interest rate r compounded continuously the amount is

$$P(t) = P \cdot e^{rt}.$$

Where r is a decimal

So if we 6% interest $r = 0.06$.