MA110. Class 21

10/12/16.

- Def. of Logarithms.
- Properties
- Solving equations involving the logarithm.

Logarithm functions.

What is a log function?

Let $a > 1$. Let

$E(x) = a^x$

the exponential function w/ base a.

The function

$\underline{\log_a(x)} = E^{-1}(x)$

is the inverse to $E(x)$.

- Thus $y = \log_a(x)$ means that $x = a^y$

- If $a = e$, then $\log_e(x) = \ln(x)$.

$\ln(x)$ is called the natural logarithm.

- If $a = 10$,

$$\log_{10}(x) = \log(x).$$

is called the common logarithm.

$$\log_{10}(1). = y = 0$$

which satisfies

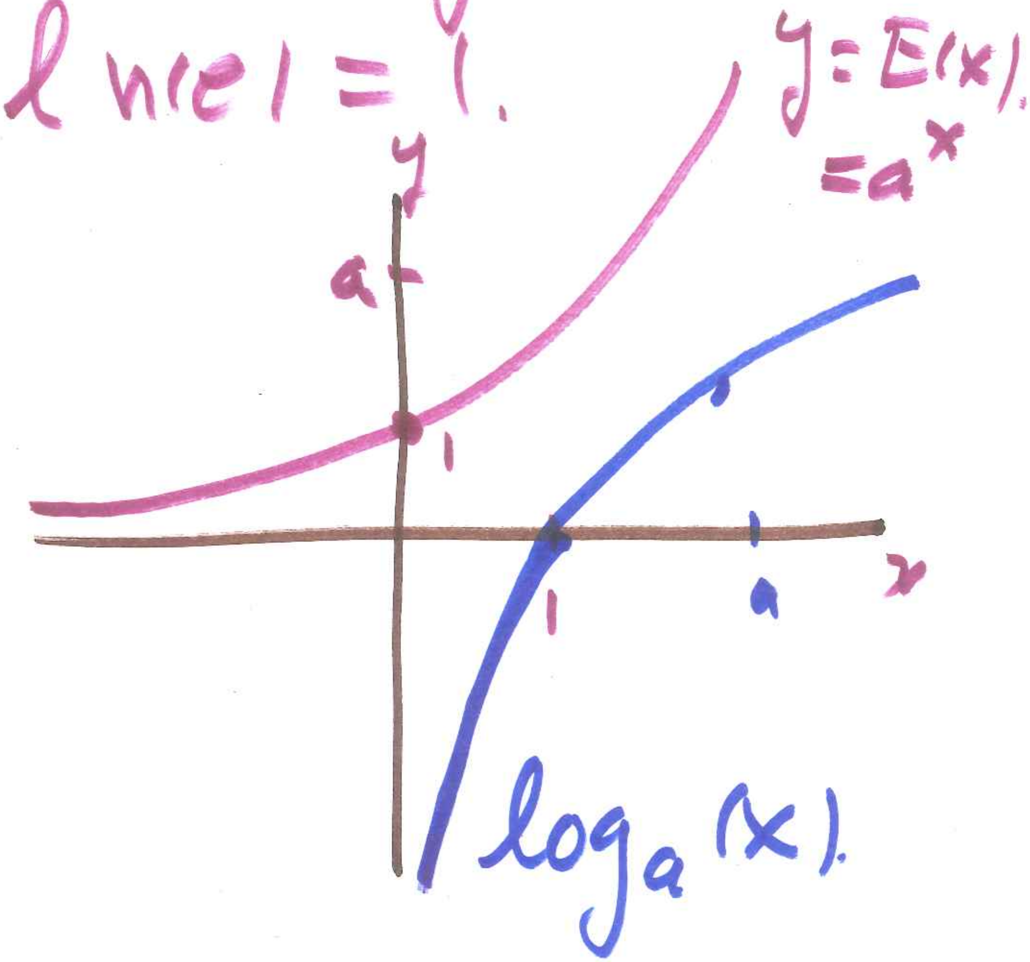
$$10^y = 1.$$

$$\ln(e) = \log_e(e)$$

$$= y$$

$$e^y = e, \quad y = 1.$$

$$\ln(e) = 1.$$



Domain of $\log_a(x)$ is $(0,\infty)$

Range is $(-\infty, \infty)$.

Compute $\log_4(2)$ and $\log_2(4)$

$y = \log_4(2)$.

is equivalent to

$4^y = 2. = \sqrt{4}$

$= \cancel{2} \; 4^{1/2}$

$y = 1/2$

$\log_4(2) = 1/2.$

$\log_2(4) = y$ with $2^y = 4$

$= 2^2$

so $\underline{y = 2}$

$$\log_5(25) + \log_{25}(1/5)$$

$y = \log_5(25)$ if

$$5^y = 25$$

or $y = 2$

$z = \log_{25}(1/5)$ if

$$\frac{1}{5} = 25^z$$

$$\frac{1}{25^{1/2}} = 25^z$$

$$25^z = 25^{-1/2}.$$

$$z = -1/2.$$

$$y + z = 2 - \frac{1}{2}$$

$$= 3/2$$

Properties.

$$\log_a(1) = 0$$

$$\log_a(a) = 1$$

$$\cancel{\log_a(\;^k)}$$

$$\log_a(a^k) = k$$

$$a^{\log_a(k)} = k.$$

Example Solve

$$\ln(1-x) = 1.$$

Recall $\ln(x) = \log_e(x)$.

$$e^{\ln(1-x)} = e^1$$

$$1-x = e$$

$$1-e = x$$

$$x = 1 - 2.7818\ldots$$

Solve

$\cancel{\log_4 (2)}$

$$\log_4 (x^2) = 2$$

$$4^{\log_4 (x^2)} = 4^2$$

$$x^2 = 16$$

$$x = \underline{+4 \text{ or } -4}$$

REEF

Solve

$$\log_2 (x^2 - x) = 1.$$

Enter solutions with a comma separating them.

$$2^{\log_2(x^2-x)} = 2$$

$$x^2 - x = 2$$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2 \text{ or } -1.$$

Find the inverse function to

$$f(x) = \ln(2x+3).$$

$$y = \ln(2x+3)$$

$$e^y = 2x+3$$

$$e^y - 3 = 2x$$

$$\frac{1}{2}(e^y - 3) = x$$

$$f^{-1}(y) = \frac{1}{2}(e^y - 3)$$

$$f^{-1}(x) = \frac{1}{2}(e^x - 3).$$

Find the inverse

of $g(x) = 1 + \log(x)$

Recall

$\log(x) = \log_{10}(x).$

Enter 10^{x} as

~~10 ^ x.~~ 10 ^ x

$\cancel{y = 1 + 10^{2}}$

$y = 1 + \log(x).$

$y - 1 = \log(x)$

$10^{y-1} = 10^{\log(x)}$

$10^{y-1} = x$

$g^{-1}(x) = 10^{x-1}.$

2nd solution

$y = 1 + \log(x).$

$$10^y = 10^{1+\log x}$$

$$10^y = 10 \cdot 10^{\log x}$$

Using rules for exponents

$$10^y = 10 \cdot x$$

or $x = \frac{1}{10} 10^y = 10^{y-1}$

$$g^{-1}(x) = 10^{x-1}.$$

Enter as 10 ^ (x - 1).