MA 110 - 10/19/16.

- Exam grades today or tomorrow.

- Midterm grades Friday or Monday.

Solve

$$2^{x+1} = 4^x.$$

- Get a common base

$$2^{x+1} = (2^2)^x = 2^{2x}.$$

- Match exponents.

$$x+1 = 2x$$

- Solve

$$1 = 2x - x$$

$$\underline{x = 1}.$$

Check.

$$2^{1+1} \stackrel{?}{=} 4^1$$

$$4 = 4 \checkmark.$$

Fancier terminology

- Apply ~~$\log_4$~~ $\log_2$

$$\log_2(2^{x+1}) = \log_2(2^{2x}).$$

$$x+1 = 2x$$

Solve

$$\frac{(3^x + 6)}{3} = 5$$

Solve for 3^x.

$$\frac{(3^x+6)}{3} \cdot 3 = 5 \cdot 3$$

$$3^x + 6 - 6 = 15 - 6$$

$$3^x = 9. = 3^2$$

$$\underline{x = 2}$$

Solve

$$5^x = 8$$

(Secretly we are looking for $\log_5 8$.)

Answer.

$$\ln(5^x) = \ln(8).$$

$$x \ln(5) = \ln(8).$$

$$x = \ln(8)/\ln(5)$$

REEF #2.

Solve

$$5^{x^2} = 25^x.$$

Solution.

• Check.

Common base

$$5^{x^2} = (5^2)^x = 5^{2x}$$

Match exponents

$$x^2 = 2x$$

$$x^2 - 2x = 0$$

$$x(x-2) = 0$$

$$x = 0, \text{ or } x = 2.$$

Recall a ~~funct~~ quantity grows exponentially if it is given by the expression

$$Pe^{kt}.$$

Suppose

$$P(1) = 2, \; P(5) = 12$$

and $P(t) = P_0 e^{kt}$.

Find P_0, k.

$$P(1) = 2 = P_0 e^{k} \quad (1)$$

$$P(5) = 12 = P_0 e^{5k} \quad (2)$$

Divide (2) by (1)

$$\frac{12}{2} = \frac{\cancel{P_0} e^{5k}}{\cancel{P_0} e^{k}}$$

$$6 = e^{5k-k}.$$

$$6 = e^{4k}.$$

$$\ln(6) = \ln(e^{4k}).$$

$$= 4k \ln(e).$$

$$4k = \ln(6).$$

$$k = \frac{1}{4}\ln(6)$$

$$\approx 0.448.$$

Find P_0.

$$\cancel{1}.\ 2 = P_0 e^{\frac{1}{4}\ln(6)}.$$

$$P_0 = 2 \cdot e^{-\frac{1}{4}\ln(6)}.$$

$$= 2 \left(e^{\ln(6)}\right)^{-1/4}.$$

$$= 2.\ 6^{-1/4}.$$

$$\approx 1.28$$

Check.

$$(1.28)\cdot e^{0.448\cdot 5}$$

A population doubles in 5 years and grows exponentially. Find k.

$$P(t+5) = 2P(t).$$

for $P(t) = P_0 e^{kt}$.

$$\cancel{P_0}\, e^{k(t+5)} = 2\cancel{P_0}\, e^{kt}.$$

$$\cancel{e^{kt}}\cdot e^{5k} = 2\cdot \cancel{e^{kt}}$$

$$e^{5k} = 2.$$

$$\ln(e^{5k}) = \ln(2).$$

$$5k\,\cancel{\ln(e)} = \ln(2).$$

$$k = \frac{1}{5}\ln(2).$$

$$M(t) = M \cdot e^{-kt}.$$

$$M(t+10) = \frac{1}{3} M(t)$$

$$\cancel{M} e^{-k(t+10)} = \frac{1}{3} \cancel{M} \cdot e^{-kt}.$$

$$\cancel{e^{-kt}} \cdot e^{-k10} = \frac{1}{3} \cancel{e^{-kt}}$$

$$\ln(e^{-k10}) = \ln(1/3).$$

$$-k10 = \ln(1/3).$$

$$k = -\frac{1}{10} \ln(1/3).$$

$$= +\frac{1}{10} \ln(3).$$

Since $\ln(1/3) = -\ln(3)$.

Solve

$$\ln(x) + \ln(x-2) = \ln(3).$$

$$e^{\ln(x(x-2))} = e^{\ln(3)}$$

$$x(x-2) = 3$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$x = 3$ or $x = -1$.

Check.

$$\ln(3) + \ln(3-2)$$

$$= \ln(3) + \cancel{\ln(1)}^{0} \checkmark$$

$$\ln(-1) + \ln(-1-2)$$

$$= \text{undefined.}$$

-1 is a false root.

Solve $\ln(x) - \ln(x+1) = \ln(2)$.

$$\ln\left(\frac{x}{x+1}\right) = \ln(2).$$

$$e^{\ln(x/x+1)} = e^{\ln(2)}$$

$$\frac{x}{x+1} = 2.$$

$$x = 2x + 2$$

$$-2 = x.$$

Check.

$$\ln(-2) - \ln(-2+1) = \text{undef.}$$

Note that -2 is a solution of

$$\ln\left(\frac{x}{x+1}\right) = \ln(2),$$

but not a solution of $\ln(x) - \ln(x+1) = \ln(2)$.

~~ln~~