MA110. F16 L25.

Exponential models

Compound interest.

$$P(t) = A(1+r/n)^{nt}.$$

A = initial amount.

r = interest rate as a decimal.

n = # of compoundings.

t = time

Exponential growth.

$$P(t) = A\, b^t \text{ or}$$

$$P(t) = A e^{kt}.$$

$k>0$ so that

$$b = e^k > 1.$$

A = initial value.

k = growth rate or interest rate for compound interest.

Exponential decay

$$M(t) = A e^{-kt} \text{ or}$$

$$M(t) = A b^t.$$

$k>0$, $0<b<1$.

REEF

$P(t) = 42\cdot 3^t$.

$$\frac{P(t+2)}{P(t)} = \frac{\cancel{42}\cdot 3^{t+2}}{\cancel{42}\cdot 3^{t}}$$

$$= \frac{\cancel{3^t}\cdot 3^2}{\cancel{3^t}} = 3^2$$

$$= 9.$$

REEF from Wed.

$\ln(x+1) - \ln(x) = \ln(2)$.

$$e^{\ln\left(\frac{x+1}{x}\right)} = e^{\ln(2)}$$

$$\frac{x+1}{x} = 2$$

$$x+1 = 2x$$

$$1 = x$$

$$x = 1$$

$\cancel{\ln x}$ $\ln(1+1) - \ln(1) = \ln(2)$ ✓

$\ln(x) - \ln(x+1) = \ln(2)$.

$$\ln\left(\frac{x}{x+1}\right) = \ln(2)$$

$$\frac{x}{x+1} = 2$$

$$x = 2x + 2$$

$$x - 2x = \cancel{2x - 2x} + 2$$

$$-x = 2$$

$$\underline{x = -2}$$

Check.

$$\ln(-2) - \ln(-2+1) \stackrel{?}{=}$$

undefined.

$$\ln\left(\frac{x}{x+1}\right) = \ln(2).$$

Let $x = -2$

$$\ln\left(\frac{-2}{-2+1}\right) = \ln(2).$$

Example. (compound interest).

Invest 1000 at 5% compounded monthly. When do we reach 2500?

Solution.

$M(t)$ = amount of money.

$$M(t) = 1000\left(1 + \frac{0.05}{12}\right)^{12t}$$

Solve

$$M(t) = ~~1000\left(1 + \frac{0.05}{12}\right)~~ = 2500.$$

Applying ln

$$\frac{1000\left(1+\frac{0.05}{12}\right)^{12t}}{1000} = \frac{2500}{1000}$$

$$\left(1+\frac{0.05}{12}\right)^{12t} = 2.5$$

$$\ln\left(\left(1+\frac{0.05}{12}\right)^{12t}\right) = \ln(2.5).$$

$$12t\,\ln\left(1+\frac{0.05}{12}\right) = \ln(2.5)$$

$$t = \frac{1}{12}\,\frac{\ln(2.5)}{\ln\left(1+\frac{0.05}{12}\right)}.$$

$t =$ 18.36 years.

In May of the 19th year.

18 years & 5 mos.

REEF. If we invest 500 at 8% annually compounded twice a year give the value after 12 years

Start w/	500
6 mos.	$500\cdot(1.04)$
1 yr	$500\cdot(1.04)^2$

12 yrs. or 24 interest payments.

$(1.04)^{24} \cdot 500$

Doubling times –

Suppose a population doubles every 7 years. In 2016 we have 1000 critters. How many do we have ~~after~~ t years after 2016?

~~$P(7) =$~~

$P(t)$ = # of critters at time t.

$P(7) = 2 \cdot P(0)$.

$P(t) = A \cdot 2^{t/7}$

$P(0) = A = 1000$

$P(t) = 1000 \cdot 2^{t/7}$.

REEF. Start w/ 300 Population triples every 6 hours. How many after t hours?

$P(t) = 300 \cdot 3^{t/6}$

Rabbits

1865 60,000

1867 2,400,000

when were they introduced?

$P(t) = A \cdot b^t$

t = # years after 1865

$P(0) = A = 60,000$

$P(2) = 60,000 \cdot b^2$

$= 2,400,000$

$b^2 = \dfrac{2,400,000}{60,000}$

$= 40.$

$b = \underline{\sqrt{40}}.$

Solve

$P(t) = 2$

$60,000 \cdot b^t = 2$

$$b^t = 2/60{,}000$$

$$\ln(b^t) = \ln\left(\frac{2}{60{,}000}\right)$$

~~t =~~

$$t\ln(b) = \ln\left(\frac{2}{60{,}000}\right).$$

$$t = \frac{\ln\left(\frac{2}{60{,}000}\right)}{\ln(\sqrt{40})}.$$

$$\approx -5.59.$$

or May 4 1859.

Exp. Regression.
- Not examined.

x	y
1	6
2	12
3	25
4	46

$$y = 3.06 \cdot (1.98)^x$$

This is an exponential model that approximates the given information.

Exponential regression

1. Enter data

$\boxed{\text{STAT}}$ $\boxed{\text{ENTER}}$ to select Edit.

Enter data in L1 and L2.

2. Perform regression

$\boxed{\text{STAT}}$ $\boxed{>}$ to select Calc. Press $\boxed{\vee}$ to reach ExpReg $\boxed{\text{ENTER}}$

Press $\boxed{\text{2nd}}$ $\boxed{\text{LIST}}$ $\boxed{\text{ENTER}}$ to select L1. Press $\boxed{,}$ $\boxed{\text{2nd}}$ $\boxed{\text{LIST}}$ $\boxed{\vee}$ to select L2.

Press $\boxed{,}$ $\boxed{\text{VARS}}$ $\boxed{>}$ $\boxed{\text{ENTER}}$ $\boxed{\text{ENTER}}$ to select Y_1. The screen should now read

ExpReg L1, L2, Y_1.

Press $\boxed{\text{ENTER}}$

3. Display graph.

Press $\boxed{\text{ZOOM}}$ Press $\boxed{\vee}$ to reach ZoomStat Press $\boxed{\text{ENTER}}$ to display graph.