

MA110. 10/26.

Sin, cos, tan.

- Ramanujan -

Spm R. Hunt Morgan

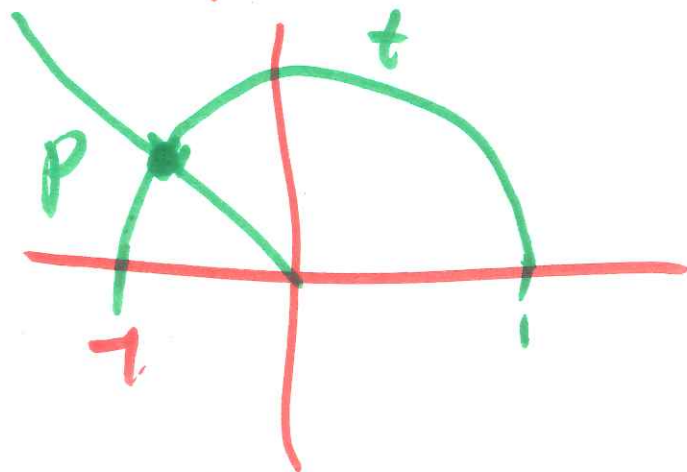
107.

- Math Excel for
MA113.

Def. of sin, cos,
tan. Draw an

angle of t radians in

Standard position.
The terminal side
crosses the unit circle
at a point P.



$$P = (\cos(t), \sin(t)).$$

$$\tan(t) = \frac{\sin(t)}{\cos(t)}.$$

$$\cot(t) = \frac{1}{\tan(t)} = \frac{\cos(t)}{\sin(t)}.$$

$$\sec(t) = \frac{1}{\cos(t)}, \csc(t) = \frac{1}{\sin(t)}.$$

Domain & range

$\sin(t)$. Domain is all real numbers

Range is $[-1, 1]$

$\cos(t)$. Domain is $(-\infty, \infty)$.

Range is $[-1, 1]$.

$$\tan(t) = \frac{\sin(t)}{\cos(t)}$$

is undefined if $\cos(t) = 0$

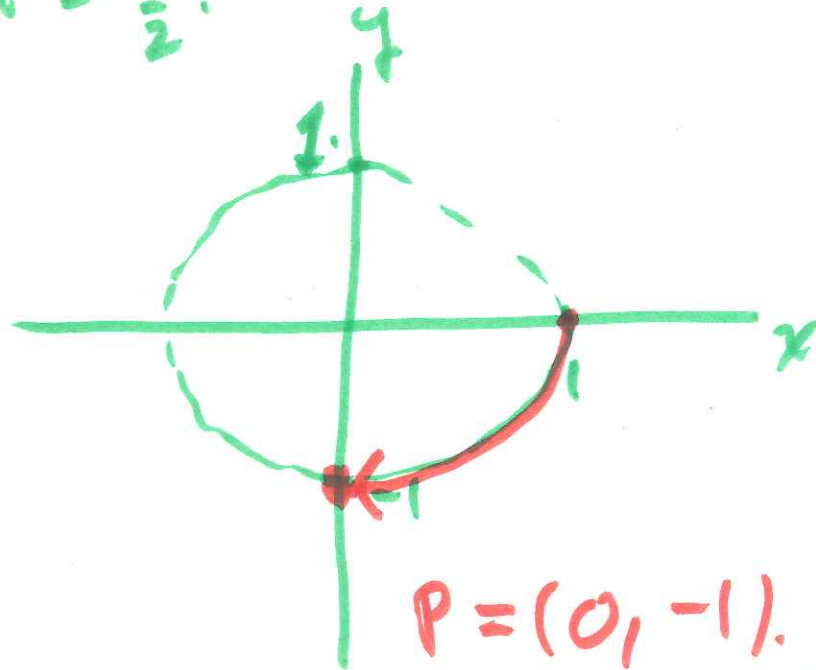
$$t = \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \dots$$

Domain is all real numbers except

$$\pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \dots$$

Range is $(-\infty, \infty)$

Example Find $\cos, \sin, \tan$ at $t = -\frac{\pi}{2}$.



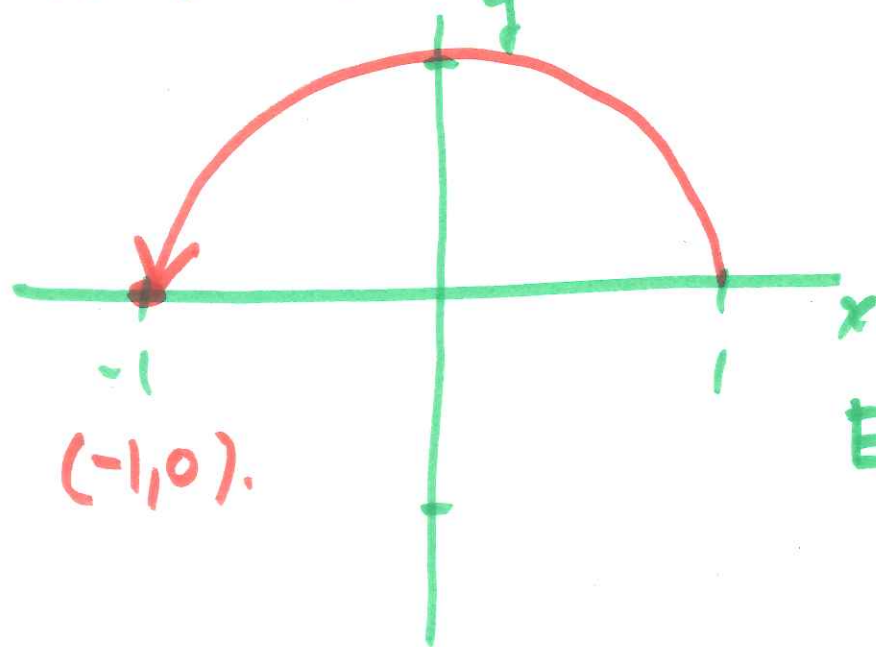
$$P = (0, -1).$$

$$= (\cos(-\frac{\pi}{2}), \sin(-\frac{\pi}{2}))$$

$$\cos(-\frac{\pi}{2}) = 0, \sin(-\frac{\pi}{2}) = -1, \tan(-\frac{\pi}{2}) = \text{undef.}$$

REG F#2.

Find $\cos(\pi)$, $\sin(\pi)$
and $\tan(\pi)$.



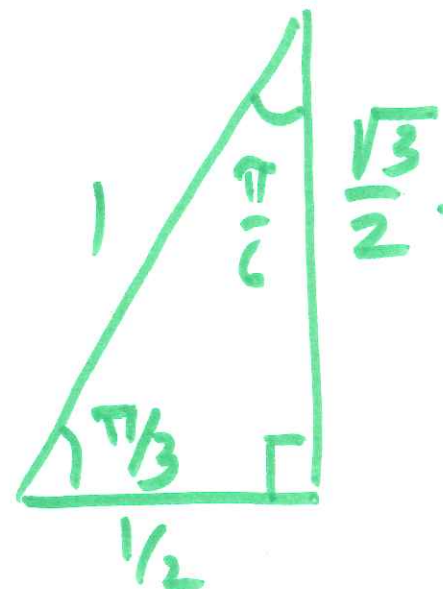
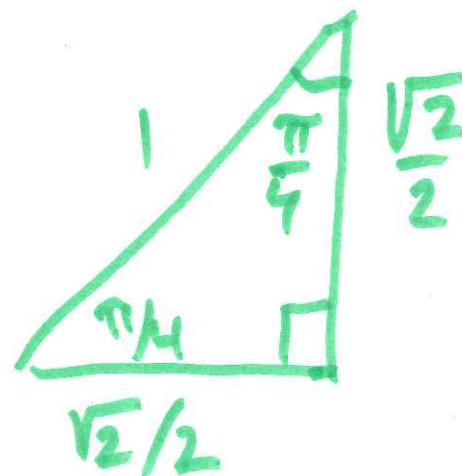
$(-1, 0)$.

$$\cos(\pi) = -1$$

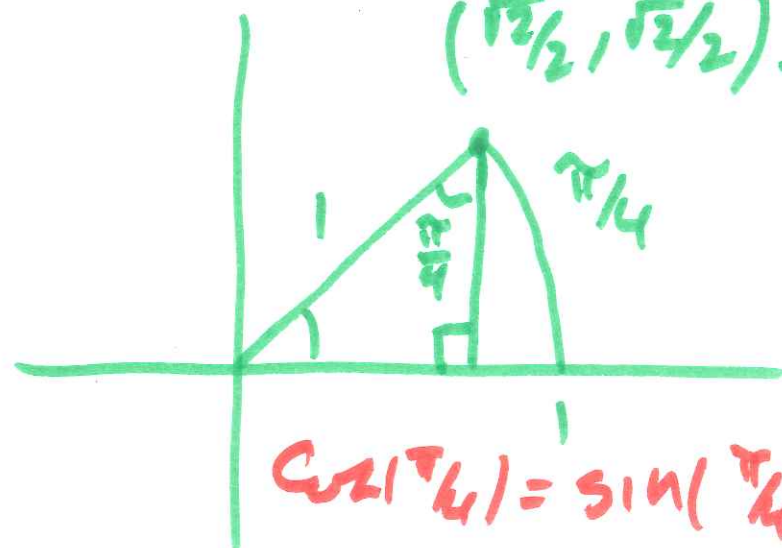
$$\sin(\pi) = 0$$

$$\tan(\pi) = \frac{0}{-1} = 0.$$

Special triangles



Ex. Find $\cos(\pi/4)$,
 $(\sqrt{2}/2, \sqrt{2}/2)$.



$$\cos(\pi/4) = \sin(\pi/4) = \frac{\sqrt{2}}{2}.$$

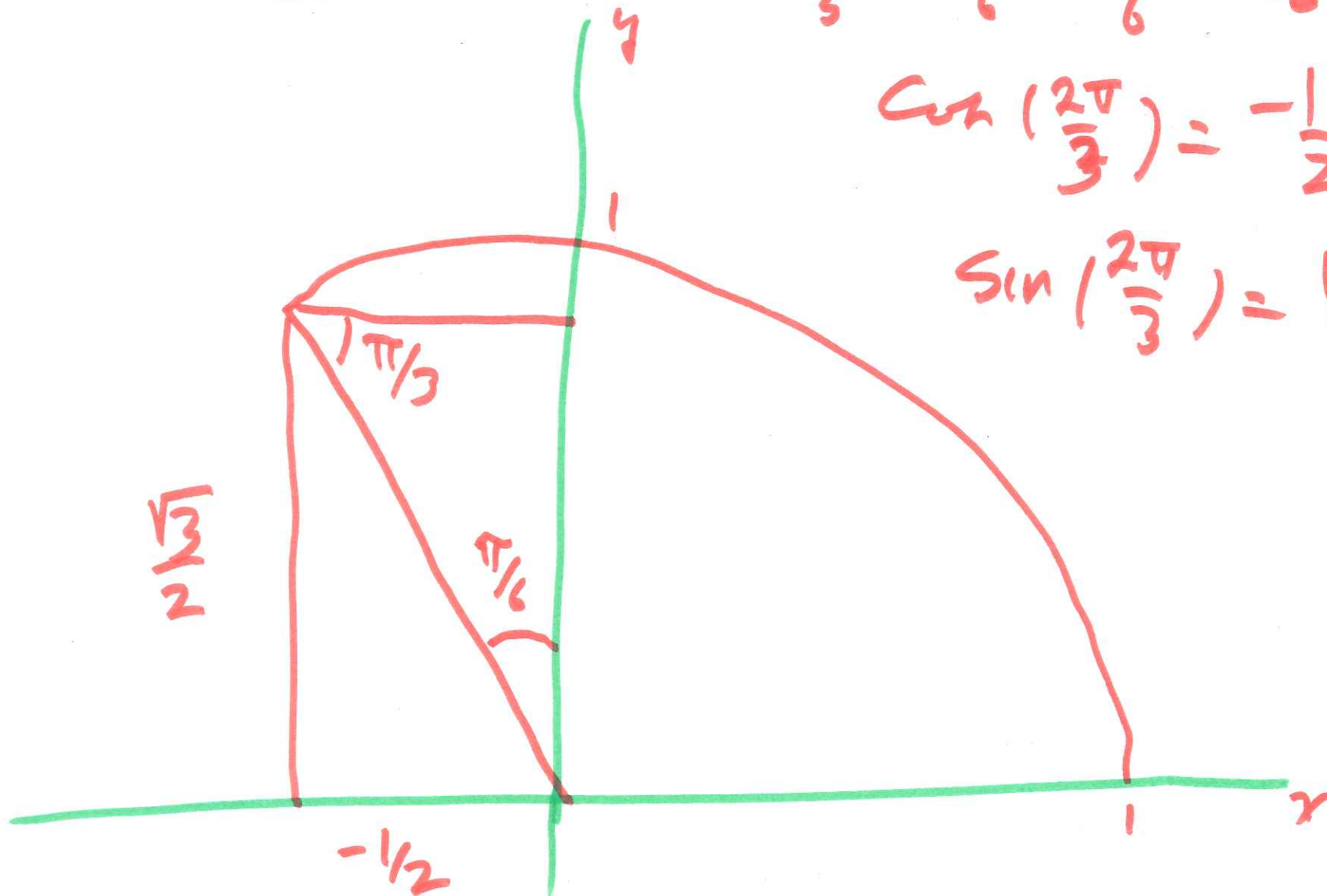
$$\tan(\pi/4) = 1$$

$$\cos(2\pi/3), \sin(2\pi/3)$$

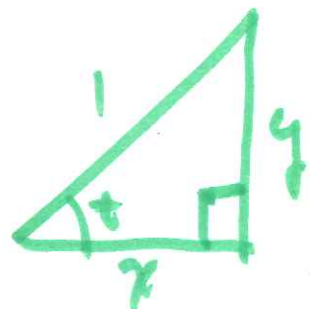
$$\frac{2\pi}{3} = \frac{4\pi}{6} = \frac{3\pi}{6} + \frac{\pi}{6}$$

$$\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$$

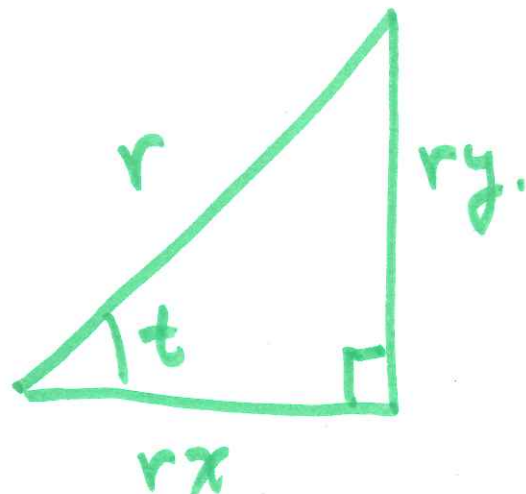
$$\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$$



Similar triangles



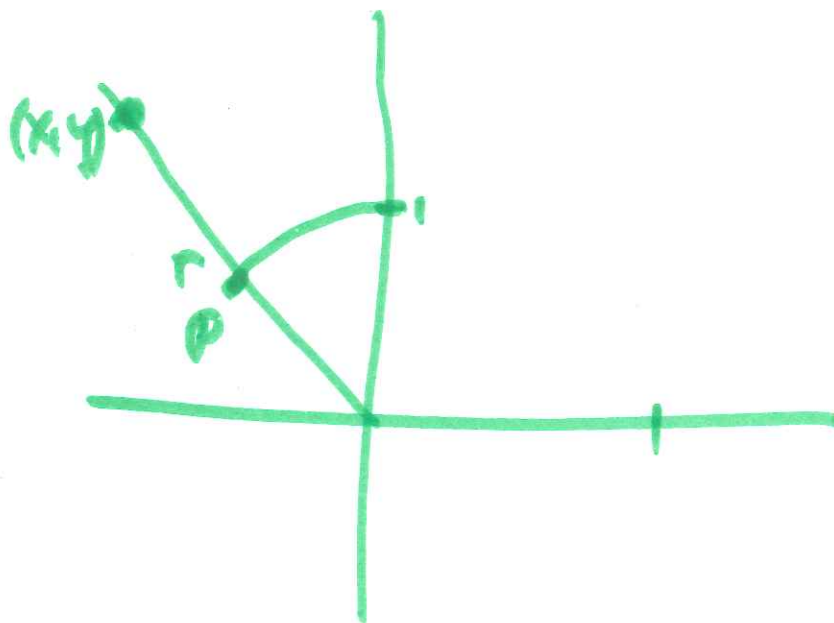
$$\sin(t) = y$$



$$\sin(t) = \frac{ry}{r} = y$$

Given a point (x, y)
on the terminal side of
an angle t .

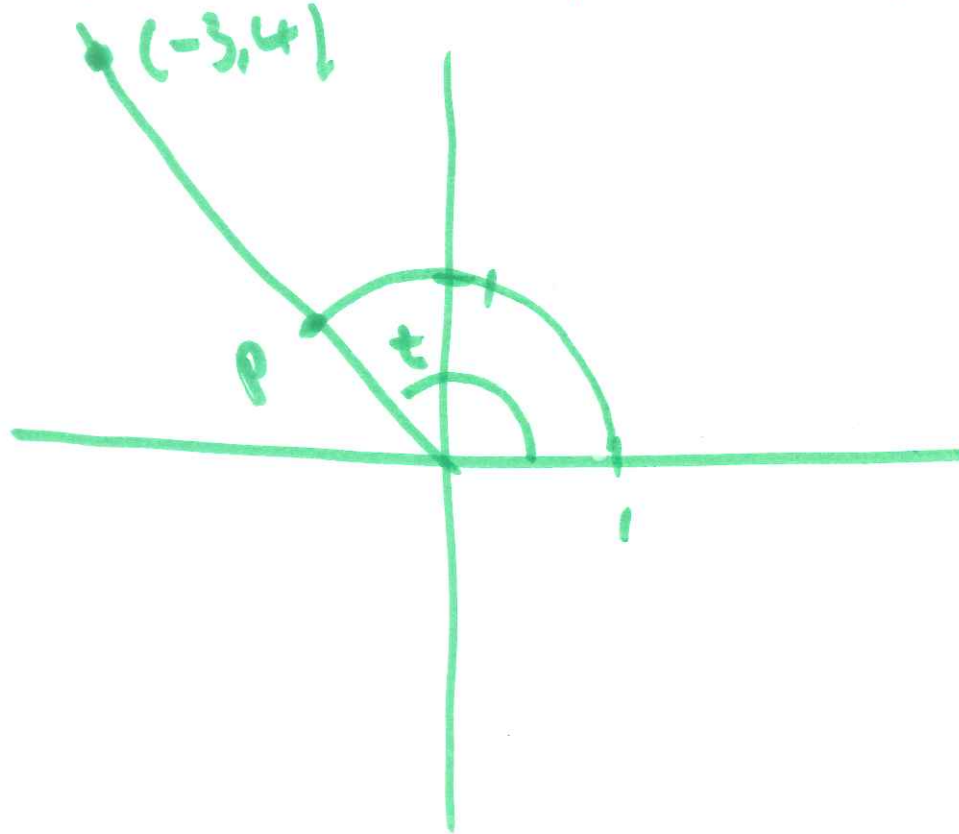
Compute $r = \sqrt{x^2 + y^2}$
this is the distance
from the origin.



$$P = \left(\frac{x}{r}, \frac{y}{r} \right) \\ = (\cos(t), \sin(t)).$$

The point $(-3, 4)$
lies on the terminal
side of an angle t .

Find $\cos(t), \sin(t)$



$$r = \sqrt{(-3)^2 + 4^2}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25} = \underline{5}$$

$$P = \left(-\frac{3}{5}, \frac{4}{5}\right)$$

$$= (\cos(t), \sin(t))$$

$$\cos(t) = -\frac{3}{5}$$

$$\sin(t) = \frac{4}{5}$$