

MA110 - 10/31.

Students moving on to
Calc. I should
consider Math Excel.

Intro question -
Find period of
 $f(x) = \sin(\pi x / 12)$.

f passes through
one period if

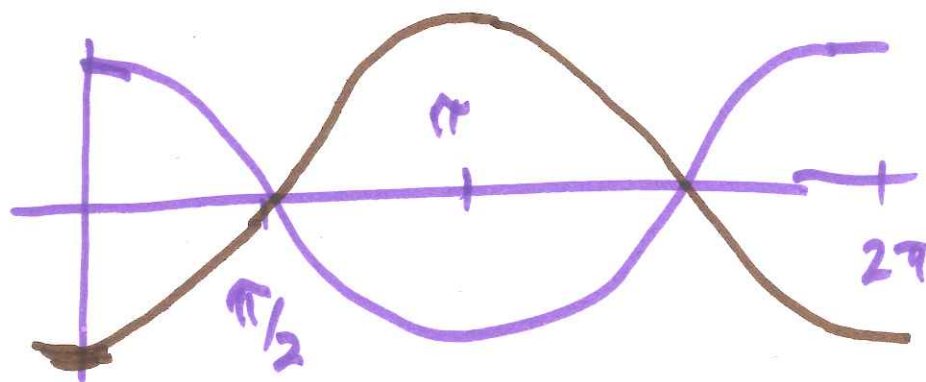
$$0 \leq \frac{\pi x}{12} \leq 2\pi$$

$$0 \leq x \leq 2\pi \cdot \frac{12}{\pi} = 24$$

Graph

$$f(x) = -\cos(x + \pi).$$

1. Graph of $\cos(x)$.
2. Shift π units to the left to obtain graph of $\cos(x + \pi)$.
3. Reflect in x -axis to obtain $-\cos(x + \pi)$.

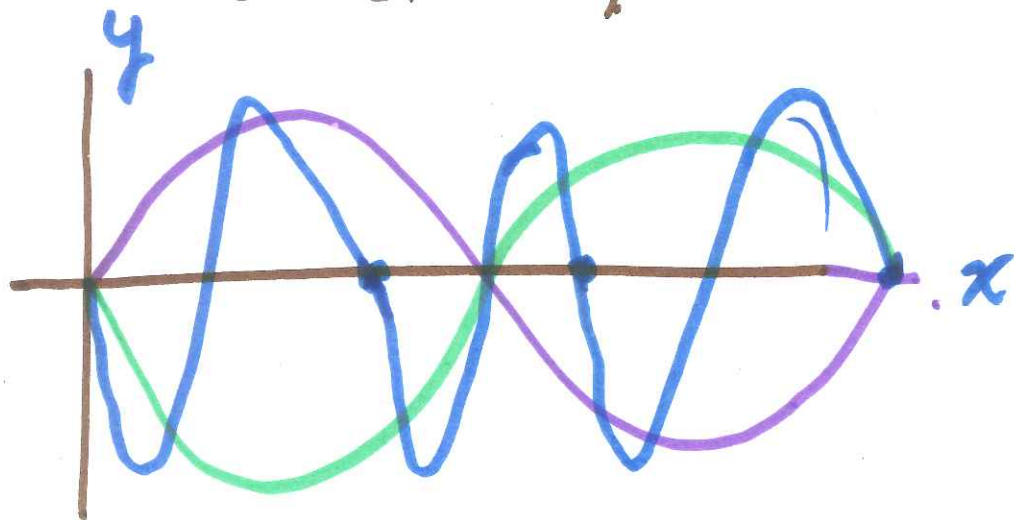


$$\cos(x) - \cos(x + \pi) -$$

$$-\cos(x + \pi) = \cos(x).$$

This is an identity - an equation
true for all x .

Graph $\sin(x)$ and transform
to obtain the graph of
~~off~~ $\sin(3x + \pi)$



• $\sin(x)$ —

$\sin(x+\pi)$ —

$\sin(3x+\pi)$ —
hor period $2\pi/3$

A. ~~SA~~ $f(x)$

Shift π units left

$$g(x) = f(x + \pi)$$

Shrink by factor 3

$$h(x) = g(3x)$$

$$= f(3x + \pi)$$

C. $f(x)$

Shrink. horiz. by 3

$$g(x) = f(3x)$$

Shift left.

$$h(x) = g(x + \pi)$$

$$= f(3(x + \pi))$$

$$= f(3x + 3\pi)$$

In general
 $f(x + \pi)$ and
 $f(3x + 3\pi)$ will
be different

But for $f(x) = \sin(x)$
 $\sin(3x + 3\pi)$
 $= \sin(3x + \pi + 2\pi)$
 $= \sin(3x + \pi).$

For functions of the
form

$$f(x) = a \sin(bx + c)$$

or $a \cos(bx + c)$
w/ $b > 0$.

$|a|$ is the amplitude

$2\pi/b$ is the period.

$-c/b$ is the phase shift.

If we write

$$a \sin(bx + c) = a \sin(b(x + \frac{c}{b}))$$

$-c/b$ represents how far to
the right we shift the

graph of $a \sin(bx)$
(or shift to left of $-\frac{c}{b}$)

Suppose daily temp.
varies between 50 to
70 degrees. Write
a function of the form
 $f(t) = A + B \sin(Ct + D)$
which models this.
Add a sine function
of amplitude 10
to 60.

$f(t) = 60 + 10 \sin(Ct + D)$.
Assume t is measured
in hours.

Want a period of 24 hrs.
 $0 \leq t \leq 24$.

To obtain $0 \leq Ct \leq 2\pi$

$$C \cdot 24 = 2\pi$$

$$C = 2\pi/24 = \pi/12.$$

$$f(t) = 60 + 10 \sin\left(\frac{\pi t}{12} + D\right)$$

If t is hours after midnight
we want the low temperature

to be $\cos \alpha t = 6$

Want

$$\frac{\pi \cdot 6}{12} + D = -\frac{\pi}{2}$$

$$D = -\pi.$$

Answer

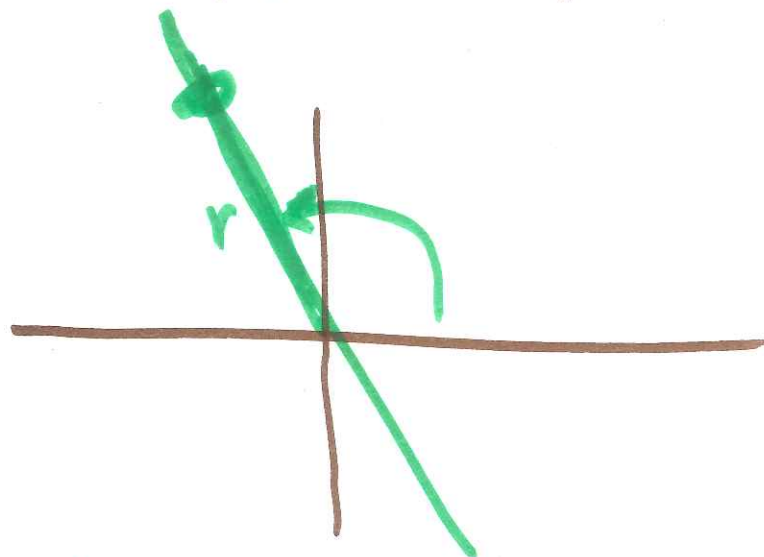
$$f(t) = 60 + 10 \sin\left(\frac{\pi t}{12} - \pi\right)$$

Review

Terminal side of angle t
lies on the line $y = -2x$.

$x < 0$. Find

$\cos(t)$, $\sin(t)$, $\tan(t)$



Pick $x = -1$, $y = -2(-1) = +2$.

$$r = \sqrt{2^2 + 1^2} = \sqrt{5}$$

$$\cos(t) = \frac{x}{r}$$

$$= -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$$

$$\sin(t) = \frac{y}{r}$$

$$= \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\tan(t) = \frac{y}{x} = \underline{\underline{-2}}$$