MA110 - #31. 11/4.

Recall

$$\tan(x) = \frac{\sin(x)}{\cos(x)} = \frac{1}{\cot(x)}$$

$$\sec(x) = \frac{1}{\cos(x)}$$

$$\csc(x) = \frac{1}{\sin(x)}$$

$$\sin^2(x) + \cos^2(x) = 1.$$

E. $\tan^2(x) - \sec^2(x)$

$$= \frac{\sin^2(x)}{\cos^2(x)} - \frac{1}{\cos^2(x)}$$

$$= \frac{\sin^2(x) - 1}{\cos^2(x)} = \frac{-\cos^2(x)}{\cos^2(x)}$$

$$= -1.$$

A: $\sin(x)\cos(x)\tan(x) + \cos^2(x)$

$$= \sin(x)\cancel{\cos(x)}\frac{\sin(x)}{\cancel{\cos(x)}} + \cos^2(x)$$

$$= \sin^2(x) + \cos^2(x) = 1.$$

C $\sin(x)\csc(x)$

$$= \sin(x)\frac{1}{\sin(x)} = 1$$

D.? graph $\cos^2(x) - \sin^2(x)$

to discover it is not 1.

-or- try values.

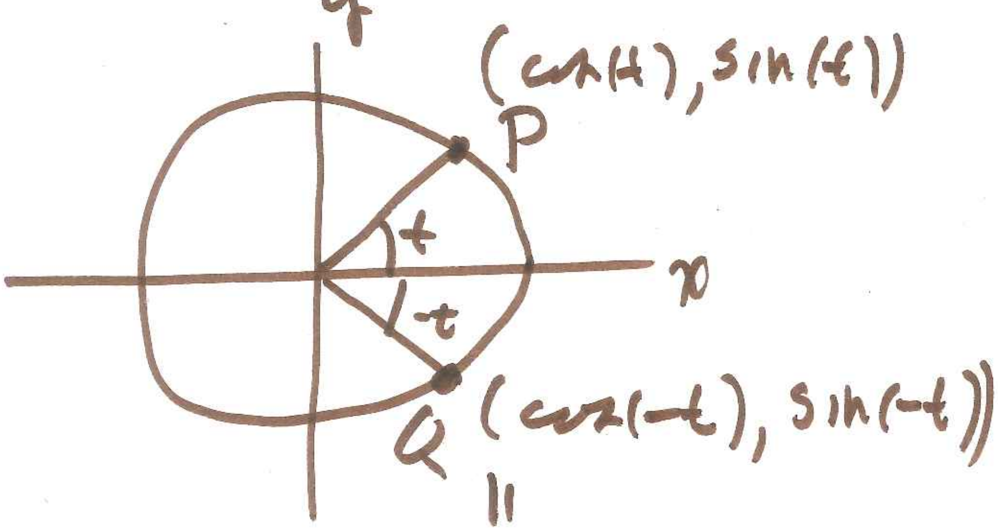
$x = \pi/4$ $\cos(\pi/4) = \sin(\pi/4)$

$= \sqrt{2}/2$, so

$$\cos^2(\pi/4) - \sin^2(\pi/4) = 0$$

Negative angle formulae

$\cos(-x) = \cos(x)$



$\parallel$

$(\cos(t), -\sin(t))$

Q is obtained by reflecting P in the x-axis.

$\cos(-t) = \cancel{\sin}\cos(t)$

$\sin(-t) = -\sin(t)$.

~~tan~~

$$\tan(-t) = \frac{\sin(-t)}{\cos(-t)} = \frac{-\sin(t)}{\cos(t)} = -\tan(t)$$

(c)

$$\sec(-x) = \frac{1}{\cos(-x)} = \frac{1}{\cos(x)} = \sec(x).$$

\- p. 516. Table of identities -

Proving identities

Prove

$\tan(x) \cdot \cot(x) = 1.$

~~tan(x) cot(x)~~

$\tan(x)\cot(x)$

$= \dfrac{\cancel{\sin(x)}}{\cancel{\cos(x)}} \cdot \dfrac{\cancel{\cos(x)}}{\cancel{\sin(x)}}$ by def of tan, cot.

$= 1.$ cancel.

Prove an identity for

$1 + 2\tan^2(x) + \tan^4(x)$

Prove an identity/cz

$$1 + 2\tan^2(x) + \tan^4(x) = \sec^4(x),$$

by def. of secant.

Solution

$$1 + 2\tan^2(x) + \tan^4(x)$$

$$= (1 + \tan^2(x))^2 \quad \text{Factoring}$$

$$= \left(1 + \frac{\sin^2(x)}{\cos^2(x)}\right)^2 \quad \text{Def of tan.}$$

$$= \left(\frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)}\right)^2 \quad \text{common denominator}$$

$$= \left(\frac{1}{\cos^2(x)}\right)^2 \quad \text{Pyth. identity}$$

Prove

$$\frac{1}{1-\sin(x)} - \frac{1}{1+\sin(x)} = 2\tan(x)\sec(x)$$

$$= \frac{1+\sin(x) - (1-\sin(x))}{(1-\sin(x))(1+\sin(x))}$$. Common denom.

$$= \frac{2\sin(x)}{(1-\sin(x))(1+\sin(x))}$$ combining like terms

$$= \frac{2\sin(x)}{1-\sin^2(x)}$$. using diff. of squares.

$$= \frac{2\sin(x)}{\cos^2(x)}$$ Py th. id.

$$= 2\tan(x)\cdot\sec(x)$$ by def. of tan & sec.

Check

$$2\tan(x)\cdot\sec(x)$$

$$= \cancel{\frac{2\sin^2(x)}{\cos}}$$

$$= 2\,\frac{\sin(x)}{\cos(x)}\,\frac{1}{\cos(x)}$$

$\sec(x) - \cos(x)$

$= \dfrac{1}{\cos(x)} - \cos(x)$ by def. of $\sec(x)$

$= \dfrac{1 - \cos^2(x)}{\cos(x)}$. common denom.

$= \dfrac{\sin^2(x)}{\cos(x)}$ pyth. id.

$= \sin(x)\tan(x)$ by def. of $\tan(x)$