MA110 - 11/7/16.

- No class tomorrow.
- Vote.
- Math Excel.

Exam on T. 11/8.*

- Experian email about alternates. Respond by Wed, 5 pm.

* Through Canvas.

REEF?

$\sin(x+y) = \sin(x) + \sin(y)$?

No.

$\sin(ax) = a\sin(x)$ - No.

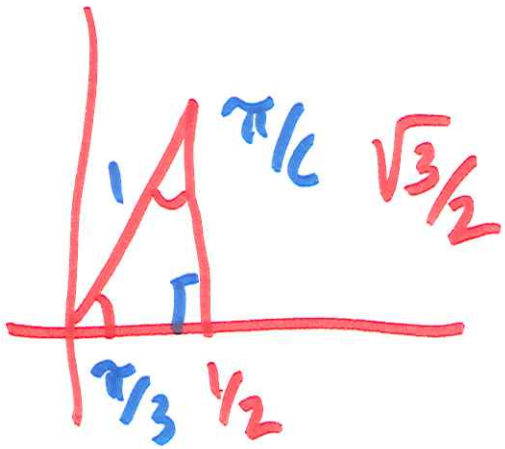
Addition formulas for sine and cosine.

$\sin(x+y)$

$= \sin(x)\cos(y) + \cos(x)\sin(y)$.

$\cos(x+y) =$

$\cos(x)\cos(y) - \sin(x)\sin(y)$



Find $\cos\left(\frac{7\pi}{12}\right)$.

Notice that

$$\frac{7\pi}{12} = \frac{(4+3)\pi}{12}.$$

$$= \frac{\pi}{3} + \frac{\pi}{4}.$$

$$\cos\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}.$$

$$\cos\left(\frac{\pi}{3}\right) = \frac{1}{2} \mid \sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}.$$

$$\cos\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \cos\left(\frac{\pi}{4}\right)\cdot\cos\left(\frac{\pi}{3}\right)$$

$$- \sin\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)$$

$$= \frac{\sqrt{2}}{2}\cdot\frac{1}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$$

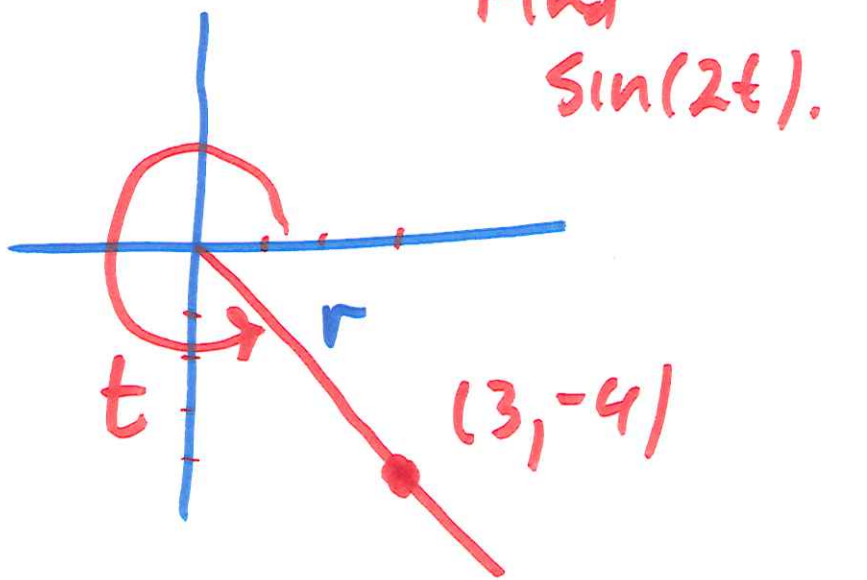
$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2}-\sqrt{6}}{4}.$$

Check $\cos\left(\frac{7\pi}{12}\right) \approx -0.259$

$$\frac{\sqrt{2}-\sqrt{6}}{4} \approx -0.259.$$

REEF #3.

Find $\sin(2t)$.



$(3, -4)$ is

$r = \sqrt{3^2 + 4^2}$

$= 5$ units from origin

$\cos(t) = 3/5$

$\sin(t) = -4/5$.

$\sin(2t) = \sin(t)\cos(t) + \cos(t)\sin(t)$

$= \frac{2}{1} \cdot 2\sin(t)\cos(t)$

$= 2 \cdot \frac{3 \cdot (-4)}{5^2} = \frac{-24}{25}$.

Find $\cos(2t)$, $\tan(2t)$

...

$\cos(2t) = \cos(t)\cos(t) - \sin(t)\sin(t)$

$= \cos^2(t) - \sin^2(t)$

$= \frac{9}{25} - \frac{16}{25} = -7/25$.

$\tan(2t) = \frac{\sin(2t)}{\cos(2t)}$

$= \frac{+24/25}{+7/25} = \frac{24}{25} \cdot \frac{25}{7}$.

$= 24/7$.

Subtraction formula

Recall

$\cos(-t) = \cos(t)$

$\sin(-t) = -\sin(t)$.

$\cos(x-y) = \cos(x+-y)$

$= \cos(x)\cos(-y)$

$- \sin(x)\sin(-y)$

$= \cos(x)\cos(y)$

$+ \sin(x)\sin(y)$.

$\cos(x-y) = \cos(x)\cos(y)$

$+ \sin(x)\sin(y)$.

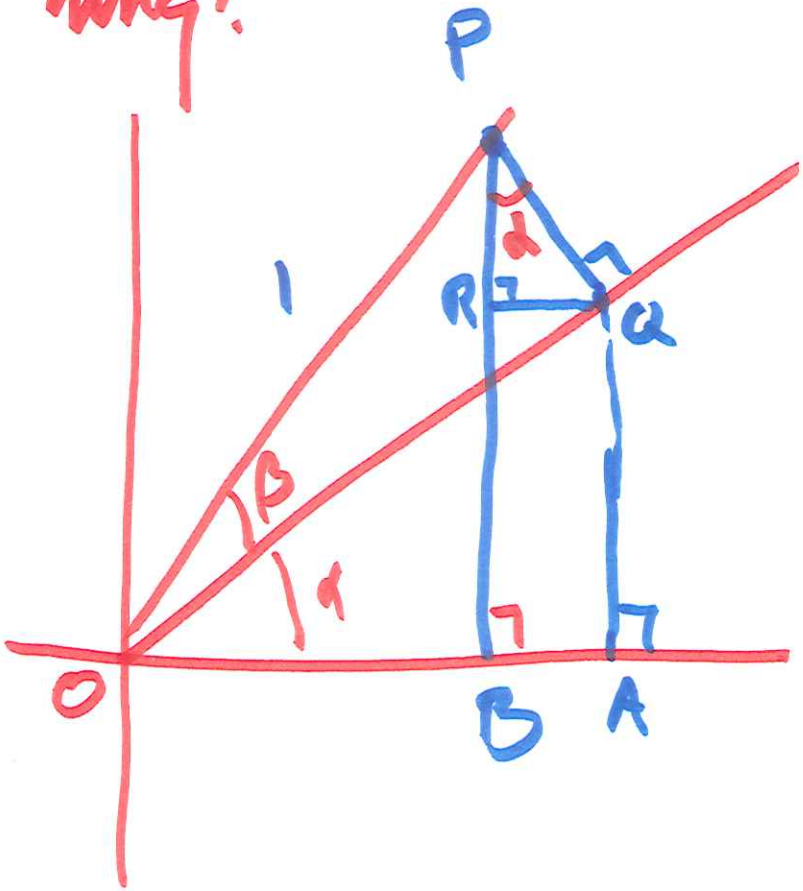
~~$\sin(x+t)$~~

$\sin(x-y) = \sin(x)\cos(y)$

$- \sin(y)\cos(x)$

Better to remember how to go from addition formula to subtraction formula, than to remember result.

Why?



Knew

$\angle OPB = \frac{\pi}{2} - \alpha - \beta$

$\angle OPQ = \frac{\pi}{2} - \beta$

$= \angle OPB + \angle RPQ$

$= \frac{\pi}{2} - \alpha - \beta + \angle RPQ.$

Simplify

$0 = -\alpha + \angle PPQ$

$\angle RPQ = \alpha$

sin(α+β)

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1. Given $|OP| = 1$

2. $\angle RPQ$

3. ΔOPQ. has hypot 1.

~~$|PQ| = \cos(\beta)$~~

~~$|OQ| = \sin(\beta)$~~

$|PQ| = \sin(\beta)$

$|OQ| = \cos(\beta)$

4. $\frac{|AQ|}{|OQ|} = \sin(\alpha)$

$|AQ| = \sin(\alpha)\cos(\beta)$

5. $\frac{|RP|}{|PQ|} = \cos(\alpha)$

$|RP| = \cos(\alpha)\sin(\beta)$.

6. $|PB| = |RP| + |AQ|$

$\sin(\alpha+\beta)$

$= \sin(\alpha)\cos(\beta)$

$+ \cos(\alpha)\sin(\beta)$.