MA110 — 11/9.

More trig identities —

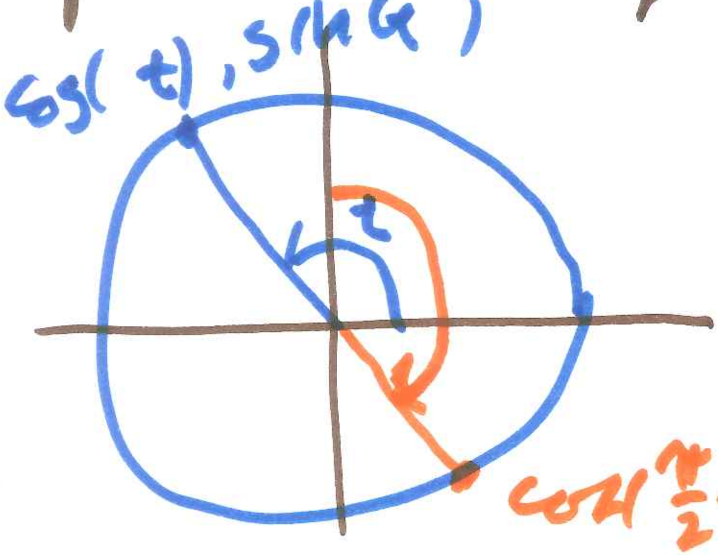
Half angle formula —

Negative angle identities

$\cos(-x) = \cos(x)$ ✓

$\sin(-x) = -\sin(x)$ ✓

Cofunction identity



Draw angle t in standard position

Draw angle $\frac{\pi}{2}-t$

Reflecting in the line $y=x$ takes

$\cos(t), \sin(t)$ to

$\cos(\frac{\pi}{2}-t), \sin(\frac{\pi}{2}-t)$.

Reflecting in $y=x$ takes

(a,b) to (b,a)

$\cos(t) = \sin(\frac{\pi}{2}-t)$ ✓

$\sin(t) = \cos(\frac{\pi}{2}-t)$ ✓

Know

$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$

Want $\cos(a+b)$.

$\cos(a+b)$ by cofun. id.

$= \sin\left(\frac{\pi}{2} - a - b\right)$

$= \sin\left(\left(\frac{\pi}{2} - a\right) + -b\right)$. addn. for sin

$= \sin\left(\frac{\pi}{2} - a\right)\cdot\cos(-b)$

$+ \cos\left(\frac{\pi}{2} - a\right)\cdot\sin(-b)$.

$= \cos(a)\cos(b)$

$+ \sin(a)\left(-\sin(b)\right)$

by neg. angle & cofun. id.

$= \cos(a)\cos(b)$

$- \sin(a)\sin(b)$.

Write $\sin(x+\pi)$ ~~as~~ as a multiple of a trig identity.

Using addn. fmla for sin:

$\sin(x+\pi)$

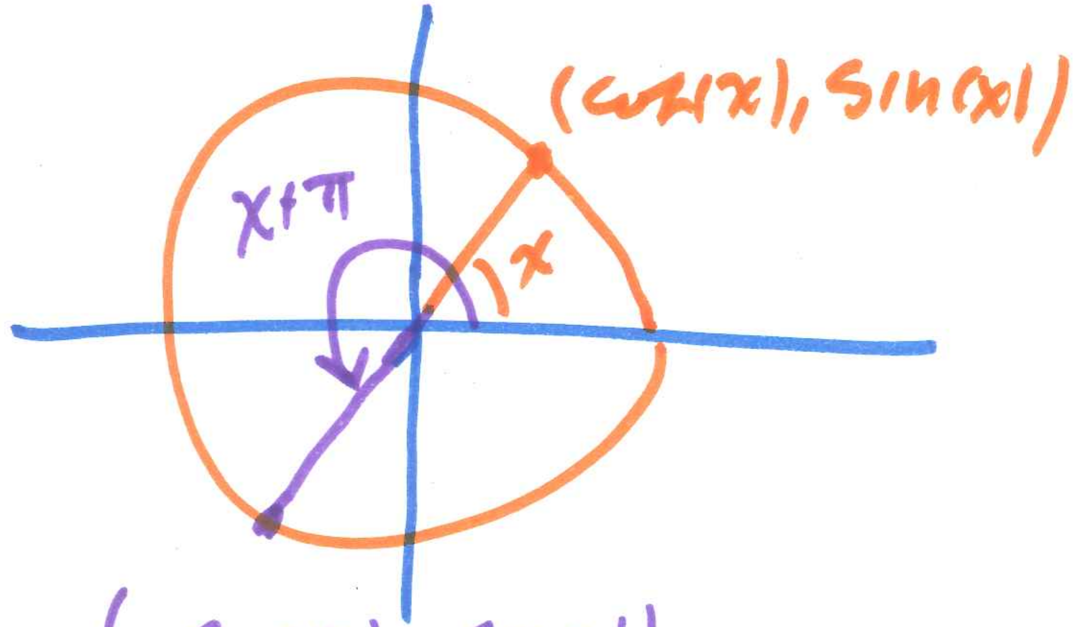
$= \sin(x)\cos(\pi) + \cos(x)\sin(\pi)$

$= -\sin(x) + 0 \cdot \cos(x)$

$= -\sin(x)$

Since $(\cos(\pi), \sin(\pi)) = (-1, 0)$

- Graph. to check or to make conjectures.



$(-\cos(x), -\sin(x))$

$= (\cos(x+\pi), \sin(x+\pi))$

$\cos(x - \frac{\pi}{2})$

$= \cos(\frac{\pi}{2} - x)$ neg. angle.

$= \sin(x)$ cof.

Double-angle formulas.

$\cos(2t)$

$= \underline{\cos^2(t) - \sin^2(t)}$ ✓

$= 1 - \sin^2(t) - \sin^2(t)$

pyth. id.

$= \underline{1 - 2\sin^2(t)}$. ✓

$= \underline{2\cos^2(t) - 1}$. ✓

$\underline{\sin(2t) = 2\sin(t)\cos(t)}$.

Half-angle formula.

$\cos(t) = 2\cos^2\left(\frac{t}{2}\right) - 1$.

$1 + \cos(t) = 2\cos^2(t/2)$.

$\frac{1+\cos(t)}{2} = \cos^2(t/2)$

$\cos(t/2) = \pm\sqrt{\frac{1+\cos(t)}{2}}$

$\sin(t/2) = \pm\sqrt{\frac{1-\cos(t)}{2}}$.

Compute $\cos\left(\frac{7\pi}{12}\right)$.

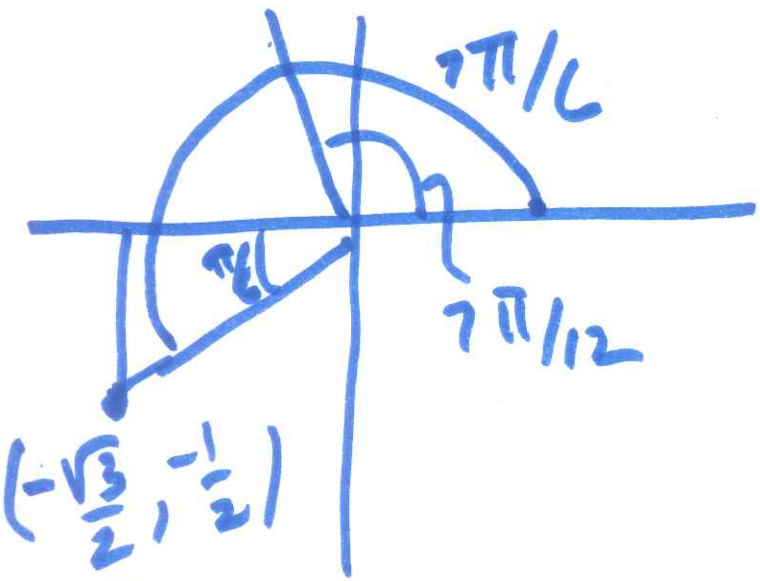
$$\cos\left(\frac{1}{2}\,\frac{7\pi}{6}\right) = \pm\sqrt{\frac{1+\cos(7\pi/6)}{2}}$$

$$= \pm\sqrt{\frac{(1-\sqrt{3}/2)}{2}\cdot\frac{2}{2}}$$

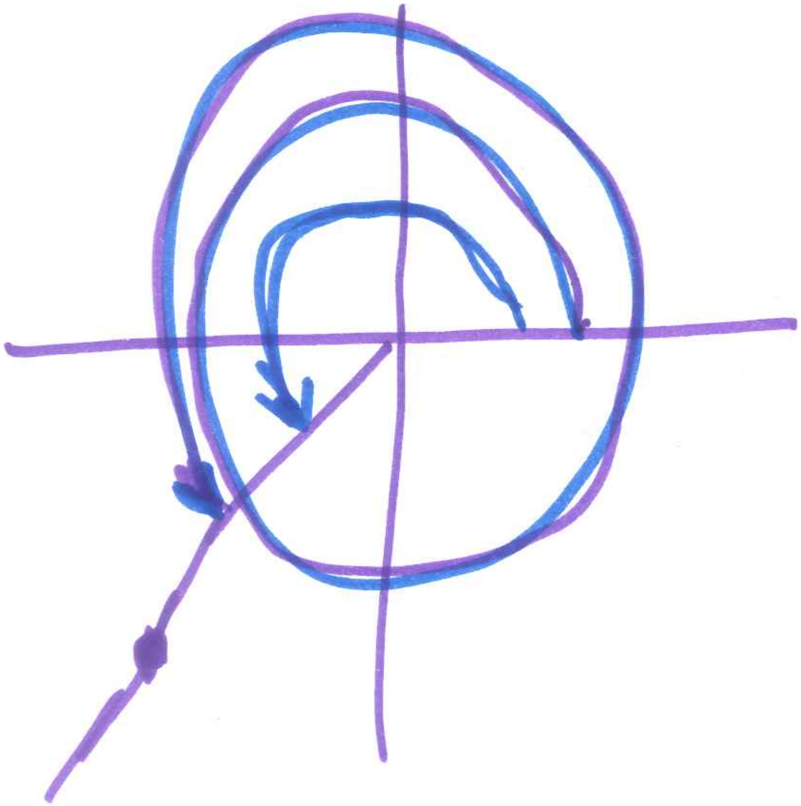
$$= \pm\sqrt{\frac{2-\sqrt{3}}{4}}$$

$$= \pm\frac{\sqrt{2-\sqrt{3}}}{2}$$

$$= -\frac{\sqrt{2-\sqrt{3}}}{2}$$ since $\frac{7\pi}{12}$ is in 2^{nd} quadrand.



$$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right) = \left(\cos\left(\frac{7\pi}{6}\right), \sin\left(\frac{7\pi}{6}\right)\right).$$



$3\pi < t < \frac{7\pi}{2}$.

$\frac{3\pi}{2} < t/2 < \frac{7\pi}{4}$.

$t/2$ in Quad. IV

$\cos(t/2) > 0$ & $\sin(t/2) < 0$.

If $\pi < t < \frac{3\pi}{2}$

then $\frac{\pi}{2} < \frac{t}{2} < \frac{3\pi}{4}$.

$\cos(t/2) < 0$ & $\sin(t/2) > 0$

Product identities –

$$2\cos(a)\cos(b)$$

$$= \cos(a)\cos(b) - \sin(a)\sin(b)$$

$$\cos(a)\cos(b) + \sin(a)\sin(b)$$

$$= \cos(a+b) + \cos(a-b).$$

$$\underline{\cos(a)\cos(b) = \frac{1}{2}\left(\cos(a+b) + \cos(a-b)\right)}$$