MA 110 / 11/14/16.

- MA113? Consider Math Excel.
- Exam T. 11/15 $7^{\underline{30}}$ - $9^{\underline{30}}$
- Review sheet and Problem set (Wkshts 23).

~~HW~~

REEF #1.

1. full turn is 2π radians.

2/3 of a circle is $\frac{2}{3} \cdot 2\pi$

$= \frac{4\pi}{3}$.

Mathskeller 10-11.

$\frac{\pi}{4} \cdot \frac{\pi}{4} \cdot \frac{\pi}{2}$ to $\frac{\pi}{3} \cdot \frac{\pi}{6} \cdot \frac{\pi}{2}$

triangles to finding cos & sin at multiples of $\frac{\pi}{6}$ & $\pi/4$.

A pp-population $P(t) = P \cdot e^{kt}$. Find a formula for the time it takes for the population to quintuple.

Want T so that

$$P(t+T) = 5 \cdot P(t).$$

for all t.

$$P \cdot e^{k(t+T)} = Pe^{kt} \cdot 5$$

$$\cancel{P}\cancel{e^{kt}} \cdot e^{kT} = \cancel{P}\cancel{e^{kt}} \cdot 5$$

$$e^{kT} = 5$$

Use the inverse to exponential function

$$\ln(e^{kT}) = \ln(5).$$

$$kT \cancel{\ln(e)} = \ln(5)$$

$$T = \frac{1}{k} \cdot \ln(5).$$

REEF #2

$P(t) = \cancel{P \cdot e} \; P \cdot 2^{t/b}$

Find P, b so that

$P(0) = 100$. $P(t+6) = 2P(t)$.

$P(0) = 100$ tells us $P = 100$

$P(0) = \cancel{100} P \cdot 2^{0/b} = 100. = P$

$P(6) = 2 \cdot P(0).$

$P(6) = P \cdot 2^{6/b}$

$= 2P(0) = 2P$

$2 = 2^{6/b}$ or $b = 6.$

$P(t) = 100 \cdot 2^{t/6}.$

<u>E</u>

Also

$100 \cdot e^{t \ln(2)/6}$

$= 100 \cdot \left(e^{\ln(2)}\right)^{t/6}$

$= 100 \cdot 2^{t/6}$

D is correct also.

An angle of $55\pi/6$ radians is in standard position. Where does the terminal side cross the unit circle?

Write $\frac{55\pi}{6} = k \cdot 2\pi + t$

with $0 \leq t < 2\pi$.

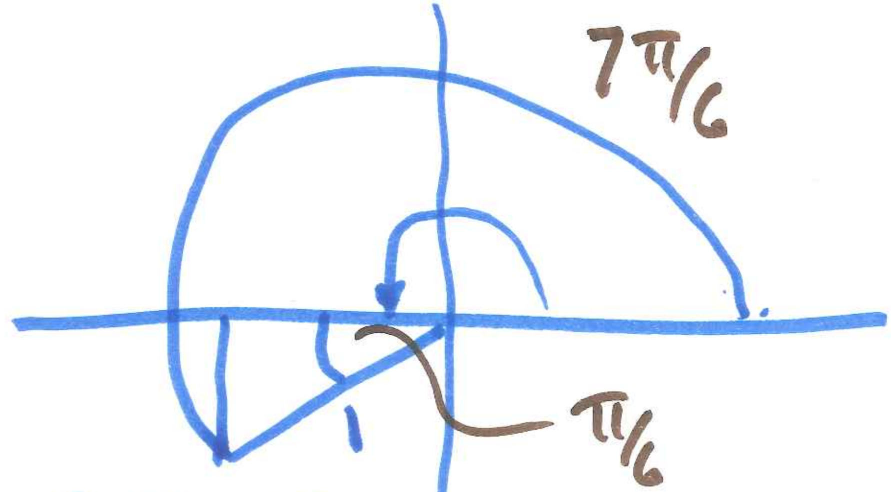
$$\frac{55\pi}{6} = \frac{48\pi}{6} + \frac{7\pi}{6}$$

$$= 8\pi + \frac{7\pi}{6}.$$

$$\frac{55\pi}{6} = \frac{54}{6}\cdot\pi + \frac{\pi}{6}$$

$$= 9\pi + \frac{\pi}{6}$$

Terminal side forms an angle $7\pi/6$ w/ the x-axis.



$$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$$

$$\|$$

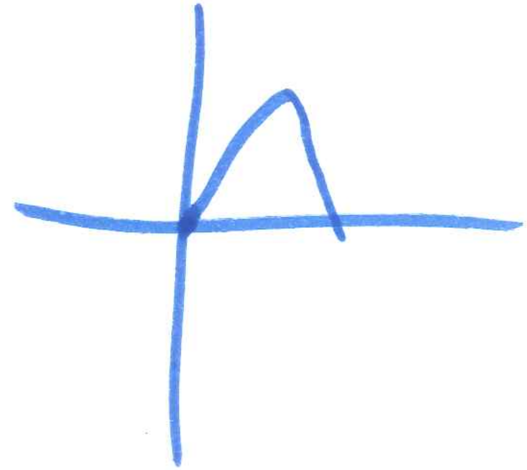
$$\left(\cos\left(\frac{7\pi}{6}\right), \sin\left(\frac{7\pi}{6}\right)\right)$$

$$= \left(\cos\left(\frac{55\pi}{6}\right), \sin\left(\frac{55\pi}{6}\right)\right)$$

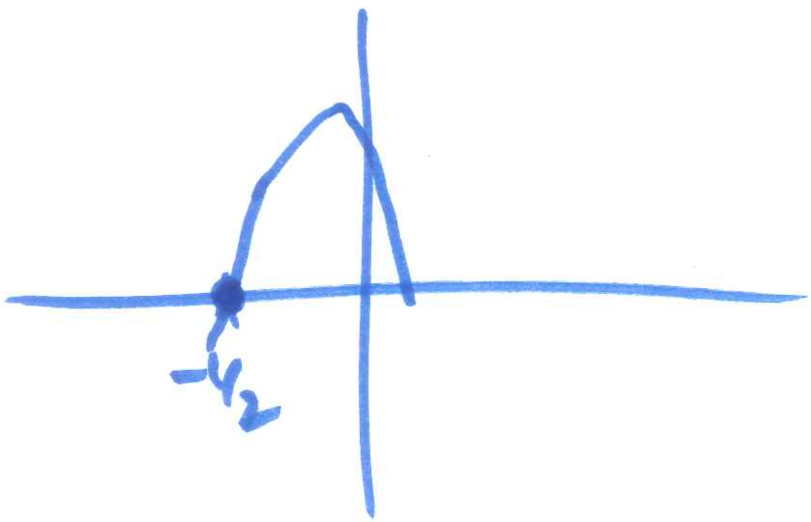
REEF #3

Find the phase shift for $f(x) = \sin(2x+1)$.

$= \sin\left(2\left(x - \frac{-1}{2}\right)\right)$



Phase shift is $-1/2$.



Use the addition formula for cosine to find an exact value $\cos(\pi/12)$.

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b).$$

Notice $\frac{\pi}{12} = \frac{4\pi}{12} - \frac{3\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$

$\cos(-\pi/4) = \sqrt{2}/2$, $\sin(-\frac{\pi}{4}) = -\frac{\sqrt{2}}{2}$.

$\cos(\frac{\pi}{3}) = \frac{1}{2}$, $\sin(\frac{\pi}{3}) = \sqrt{3}/2$.

$$\cos\left(\frac{\pi}{12}\right) = \cos\left(\frac{\pi}{3} + -\frac{\pi}{4}\right)$$

$$= \cos\left(\frac{\pi}{3}\right)\cos\left(-\frac{\pi}{4}\right) - \sin\left(\frac{\pi}{3}\right)\sin\left(-\frac{\pi}{4}\right).$$

$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\,\frac{\sqrt{2}}{2}$$

$$= \frac{\sqrt{2} + \sqrt{3}\cdot\sqrt{2}}{4}$$

$$= \frac{\sqrt{2} + \sqrt{6}}{4}.$$

Prove

$\cancel{\csc^2(x) - \cos^2(x)\csc^2(x)}$

$$\csc^2(x) - \cos^2(x)\csc^2(x) = 1$$

Write in terms of sin, cos

$$\frac{1}{\sin^2(x)} - \frac{\cos^2(x)}{\sin^2(x)} \stackrel{?}{=} 1$$

$$= \frac{1}{\sin^2(x)}\left(1 - \cos^2(x)\right) \qquad \text{Alg.}$$

$$= \frac{1}{\cancel{\sin^2(x)}}\cancel{\sin^2(x)}. \qquad \text{Pyth. id.}$$

$$= 1. \qquad \text{Alg.}$$