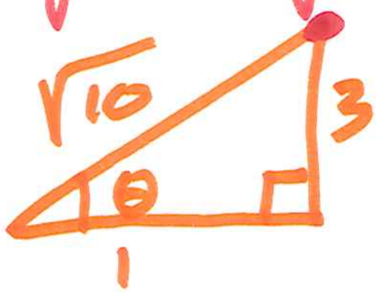
MA110 – 11/16/16.

Final exam – CB106
12/12 6 pm, Monday

Solving trig equations



$$\sin(\theta) = \frac{3}{\sqrt{10}} = \frac{3\sqrt{10}}{10}$$

what is θ?

Find **all** solutions of $\sin(\theta) = 3/\sqrt{10}$.

Useful identities

$\cos(x + 2\pi k) = \cos(x)$

$\sin(x + 2\pi k) = \sin(x)$

$\tan(x + k\pi) = \tan(x)$

$\cos(-x) = \cos(x)$ ✓

$\sin(\pi - x) = \sin(x)$ ✓

Find solutions of

$\tan(x^2) = 1$ for x

in $[-2, 2]$

$\pm\sqrt{\pi/4}, \pm\sqrt{\frac{5\pi}{4}}$.

Solve

$2\sin(x)\tan(x) - \sec(x) = 0$

$$2\frac{\sin^2(x)}{\cos(x)} - \frac{1}{\cos(x)} = 0$$

$2\sin^2(x) - 1 = 0$

$\sin^2(x) = 1/2$.

$\sin(x) = \pm\sqrt{1/2}$

$= \pm\frac{\sqrt{2}}{2}$.

$x = \frac{\pi}{4} + \cancel{2}k\pi$

$\frac{3\pi}{4} + 2k\pi$

$\frac{5\pi}{4} + 2k\pi$

$\frac{7\pi}{4} + 2k\pi$

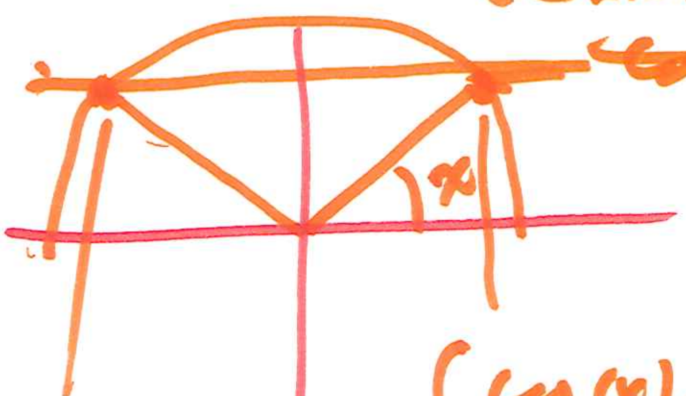
$\sin(\pi - x)$

$= \underbrace{\cancel{\sin\pi}}_{0} \cos(-x)$

$\quad + \cos(\pi)\sin(-x)$

$= -1 \cdot (-\sin(x))$

$= \sin(x)$



~~(cos(x),~~

~~(cos(π−x))~~

$(\cos(x), \sin(x))$

$(\cos(\pi - x), \sin(\pi - x))$

Solve

$\tan(x) = 2$

Find all solutions

One solution

Apply inverse function $\tan^{-1}$ or arctan.

$x = \tan^{-1}(2)$

is a solution in

$(-\pi/2, \pi/2)$

$x \approx 1.1071$

from Calculator

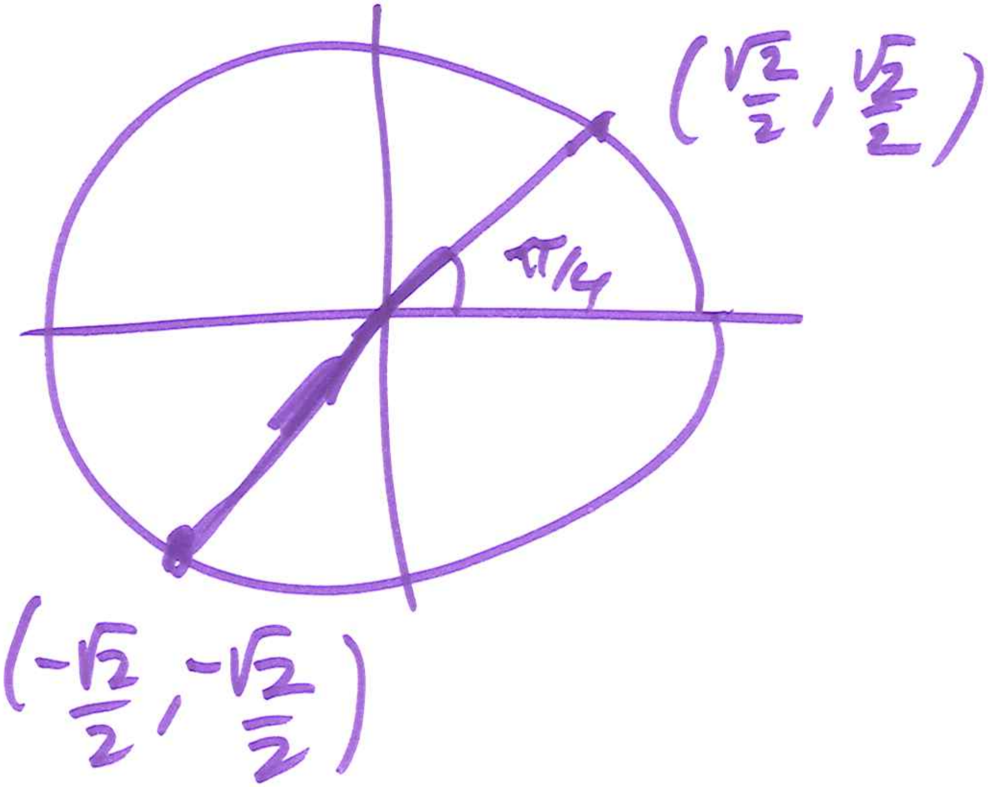
All solutions are of the form

$x \approx 1.1071 + k\pi$

$k = 0, \pm 1, \pm 2, \pm 3 \ldots$

REEF #2.

Find all solutions of $\tan(x) = 1$.



$\tan\left(\frac{\pi}{4}\right) = 1$, so

$\frac{\pi}{4} = \tan^{-1}(1)$ is one solution.

$$\frac{\pi}{4} + k\pi \frac{4}{4} = \frac{\pi + 4k\pi}{4}$$

$k = 0, \pm 1, \pm 2, \ldots$

Solving

$\sin(x) = c$

Find one solution using $\sin^{-1}$

$x = \sin^{-1}(c)$

Recall

$\sin(\pi - x) = \sin(x)$.

$\pi - \sin^{-1}(c)$ is a 2^{nd} solution.

Periodicity

$\sin^{-1}(c) + 2k\pi$

$\pi - \sin^{-1}(c) + 2k\pi$

$k = 0, \pm 1, \pm 2 \ldots$

Solve

$\sin(x) = \frac{1}{2}$.

$x = \sin^{-1}(\frac{1}{2}) = \pi/6$

$\pi - \frac{\pi}{6} = \frac{5\pi}{6}$

$\frac{\pi}{6} + 2k\pi, \quad \frac{5\pi}{6} + 2k\pi$

$k = 0, \pm 1, \pm 2, \ldots$

Solve

$\cos(x) = .4$

$x = \cos^{-1}(0.4)$.

≈ 1.16

$\cos(-x) = \cos(x)$.

$x \approx -1.16$ is a 2^{nd} solution.

All solutions

$1.16 + 2k\pi$

$-1.16 + 2k\pi$,

$k = 0, \pm 1, \pm 2, \ldots$

REE P #3

Solve

$\cos(x) = 2$

$x = \cos^{-1}(2)$

$\longrightarrow$ Error.

Range of cosine is $[-1, 1]$ so $\cos(x) = 2$ has no solution.

Recall that to solve

$$x^4 + 2x^2 + 7 = 0$$

we view this as a quadratic equation for $u = x^2$

$$(x^2)^2 + 2(x^2) + 7 = 0$$

$$\vdots$$

Find all solutions of $\tan(x^2) = 1$

Solutions are x with

$$x^2 = \frac{\pi}{4} + k\pi$$

$$k = 0, \pm 1, \pm 2, \ldots$$

So

$$x = \pm\sqrt{\frac{\pi}{4} + k\pi}$$

$$k = 0, \pm 1, \ldots$$

Find solutions of

$\tan(x^2) = 1$. with

x in $[-2, 2]$

$x = \pm\sqrt{\frac{\pi}{4}}, \pm\sqrt{\frac{5\pi}{4}}$.

Solve

$2\sin(x)\tan(x) - \sec(x) = 0$

$2\frac{\sin^2(x)}{\cos(x)} - \frac{1}{\cos(x)} = 0$

$2\sin^2(x) - 1 = 0.$

$\sin^2(x) = \frac{1}{2}$

$\sin(x) = \pm\frac{1}{\sqrt{2}} = \pm\frac{\sqrt{2}}{2}.$

$\sin(x) = +\frac{\sqrt{2}}{2}$ means

$x = \frac{\pi}{4} + 2k\pi$

$\frac{3\pi}{4} + 2k\pi$

$\sin(x) = -\frac{\sqrt{2}}{2}$ (

$x = -\frac{\pi}{4} + 2k\pi$

$\frac{5\pi}{4} + 2k\pi$

$k = 0 \pm 1, \pm 2 \ldots$