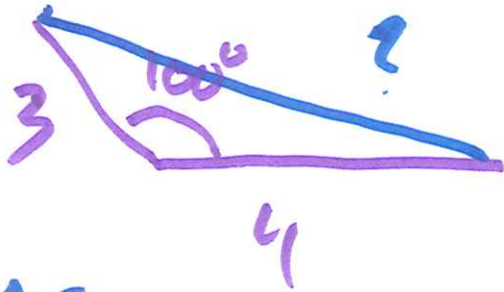
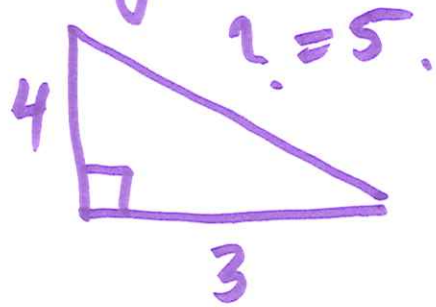


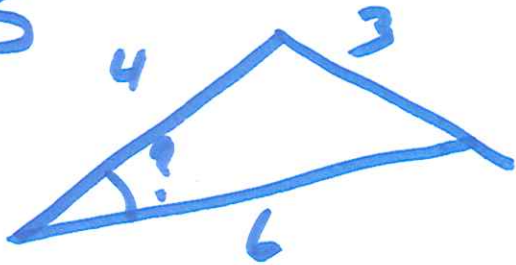
MA 110 - L37 11/18.

Law of Cosines -

Pythagorean's thm.



SAS



SSS

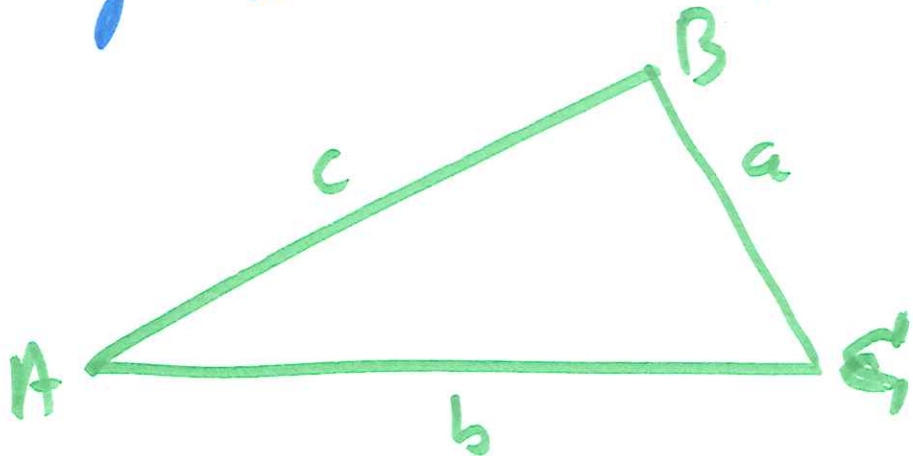
Naming convention for triangles.

A, B, C for angles

a, b, c for sides

angle A is opposite side a

angle B --- side b



Law of cosines.

$$c^2 = a^2 + b^2 - 2ab \cos(C)$$

If C is a right angle,
 $\cos(C) = 0$, & this
becomes Pythagoras.

A triangle has sides
of length 2, 3, 4.

Find the measure of
the smallest angle.

Solution



Law of cos. Given an equation
relative a, b, c, C

$$c^2 = a^2 + b^2 - 2ab \cos(C)$$

$$4 = 16 + 9 - 2 \cdot 3 \cdot 4 \cos(C)$$

$$4 = 25 - 24 \cos(C)$$

$$4 - 25 = -24 \cos(C).$$

$$-21 = -24 \cos(C)$$

$$\cos(C) = \frac{21}{24}.$$

$$C = \cos^{-1}(21/24).$$

$$\approx 0.5054 \text{ rad.}$$

$$\text{or } \approx 29^\circ$$

Other forms of
Law of Cos.

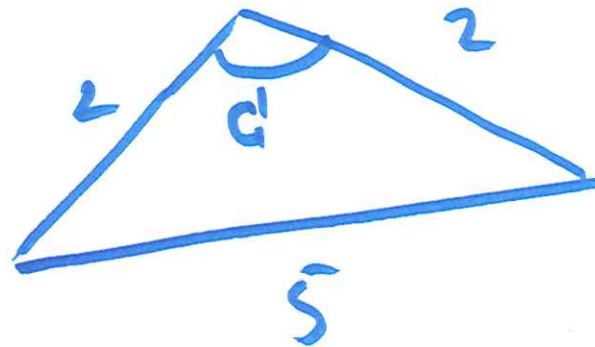
$$c^2 = a^2 + b^2 - 2ab \cos(C)$$

$$\begin{array}{l} a \rightarrow b \quad b \rightarrow c, \quad c \rightarrow a \\ A \rightarrow B \quad \quad \quad C \rightarrow A \end{array}$$

$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

$$b^2 = c^2 + a^2 - 2ac \cos(B)$$

REEF #2.



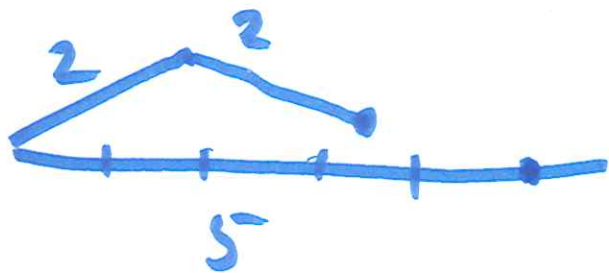
$$25 = 4 + 4 - 2 \cdot 2 \cdot 2 \cdot \cos(C).$$

$$25 = 8 - 8 \cos(C)$$

$$17 = -8 \cos(C)$$

$$\cos(C) = -\frac{17}{8}$$

Range of $\cos$ is
 $[-1, 1]$.



No such triangle.

SAS

Given 2 sides & the angle between them.

Find ~~angle~~ Side

Suppose a triangle has
 $a=10, b=20, C=60^\circ$

Find c .

$$c^2 = a^2 + b^2 - 2ab \cdot \cos(C)$$

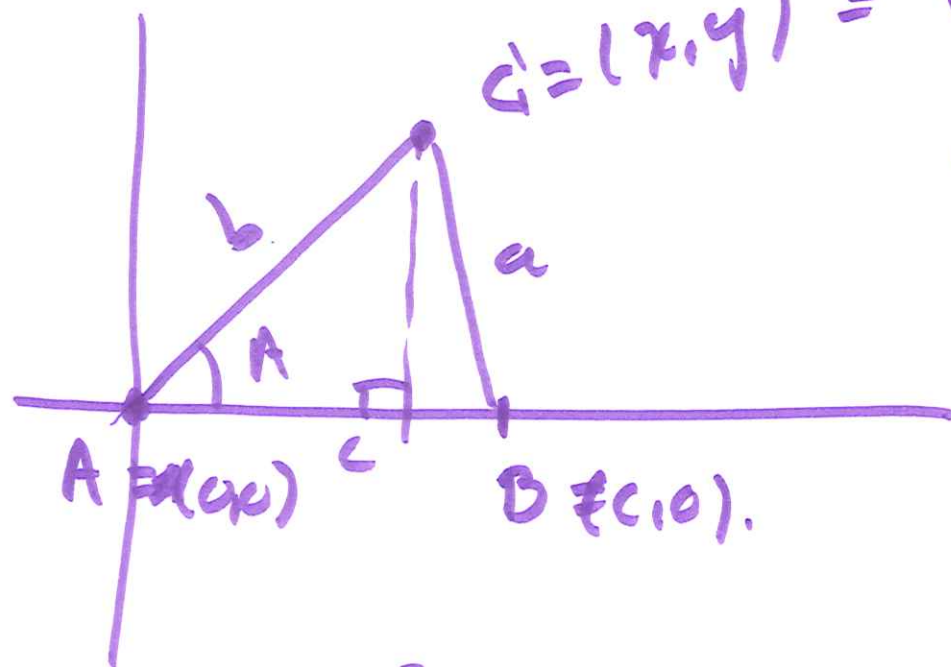
$$= 10^2 + 20^2 - 2 \cdot 10 \cdot 20 \cdot \cos(60)$$

$$= 100 + 400 - 400 \cdot \frac{1}{2}$$

$$= 300$$

$$c = \sqrt{300}$$

Why?



$$C = (x, y) = (b \cos(A), b \sin(A))$$

$$a^2 = (c - b \cos(A))^2 + (b \sin(A))^2$$

$$= c^2 + b^2 \cos^2(A) - 2bc \cos(A)$$

$$+ b^2 \sin^2(A)$$

$$= c^2 + b^2 (\cos^2(A) + \sin^2(A)) - 2bc \cos(A)$$

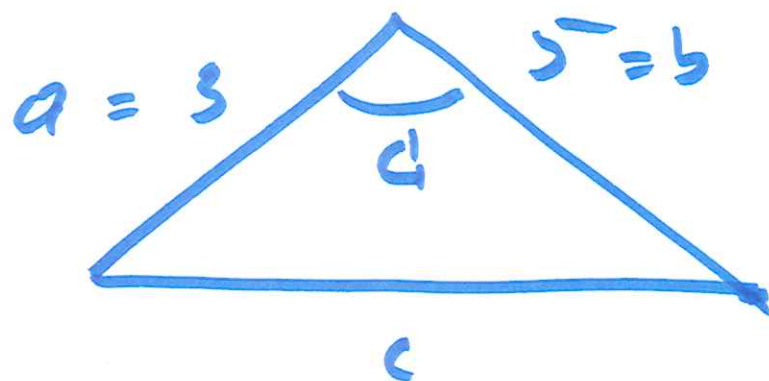
$$- 2bc \cos(A)$$

Question $a^2 = ?$

Dist. formula

$$a^2 = \text{distance}^2 \text{ from } (c, 0) \text{ to } (b \cos(A), b \sin(A))$$

$$a^2 = b^2 + c^2 - 2bc \cos(A)$$



$$c^2 = a^2 + b^2 - 2ab \cos(C).$$

$$= 3^2 + 5^2 - 2 \cdot 3 \cdot 5 \cdot (-0.8)$$

$$= 9 + 25 + 30(0.8)$$

$$= 34 + 24 = 58$$

$$c = \sqrt{58}.$$