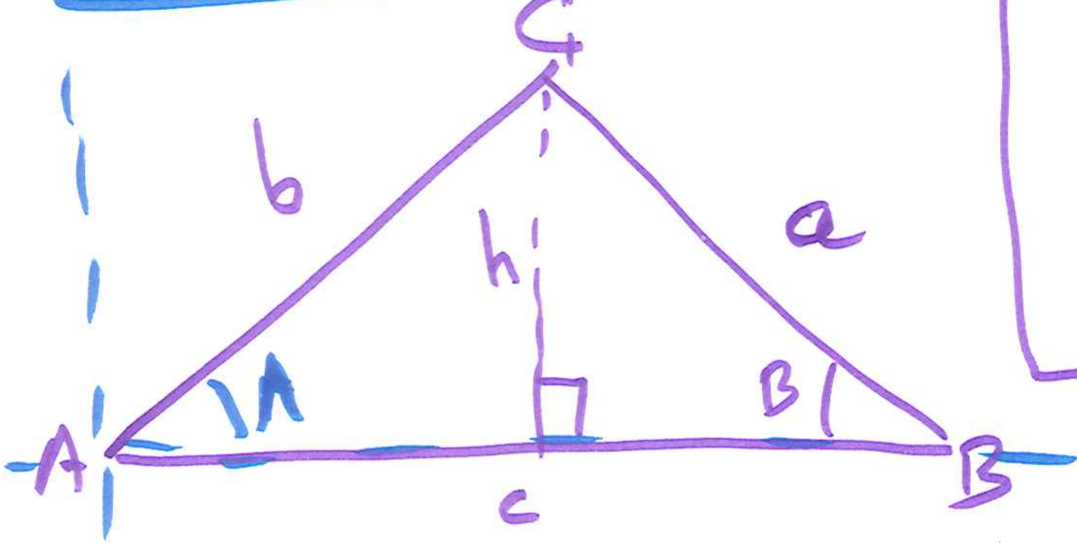
MA 110. 21 Nov.

Law of Sines

Final · Monday
6-8 pm in CB106
Cumulative
- More guidance on review is coming.



$$\sin(A) = \frac{h}{b}$$

$$h = b\sin(A).$$

$$\sin(B) = \frac{h}{a}$$

$$h = a\sin(B).$$

!

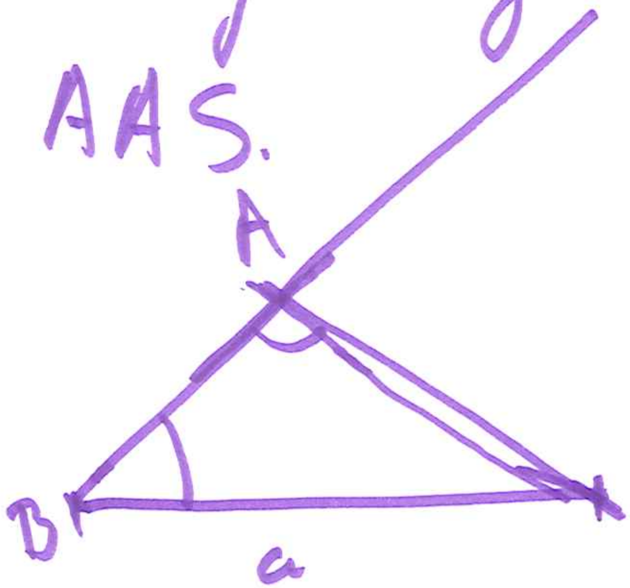
Thus

$$a\sin(B) = b\sin(A)$$

$$\boxed{\text{or } \frac{\sin(B)}{b} = \frac{\sin(A)}{a} = \frac{\sin(C)}{c}}$$

Solving triangles

AAS.



Find remaining sides & angles.

~~E~~ <u>Example</u>

$B = A = 30^\circ$ and $a = 10$. Find remaining sides and angles.

Use Law of sines.

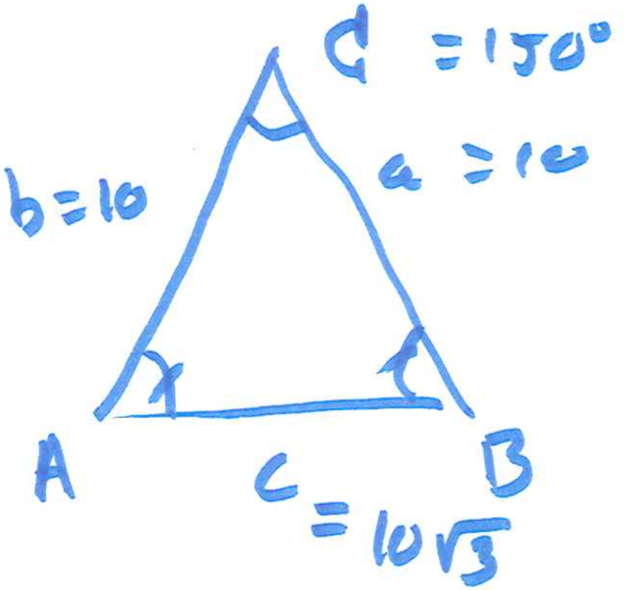
$$\frac{\sin(B)}{b} = \frac{\sin(A)}{a}$$

Know A, B, a. Solve for b.

$$b = \frac{a}{\sin(A)} \cdot \sin(B).$$

$$= \frac{10}{\cancel{\sin(30^\circ)}} \cdot \cancel{\sin(30^\circ)}.$$

$$= \frac{10}{}$$



Angle $C' = 180 - A - B$

$C' = 180 - (30 + 30)$

$= \underline{120}$

Use Law of Sines

$$\frac{\sin(C')}{c} = \frac{\sin(A)}{a}$$

$$c = \frac{a \cdot \sin(C')}{\sin(A)}$$

$$= \frac{10 \cdot \sin(120)}{\sin(30)}$$

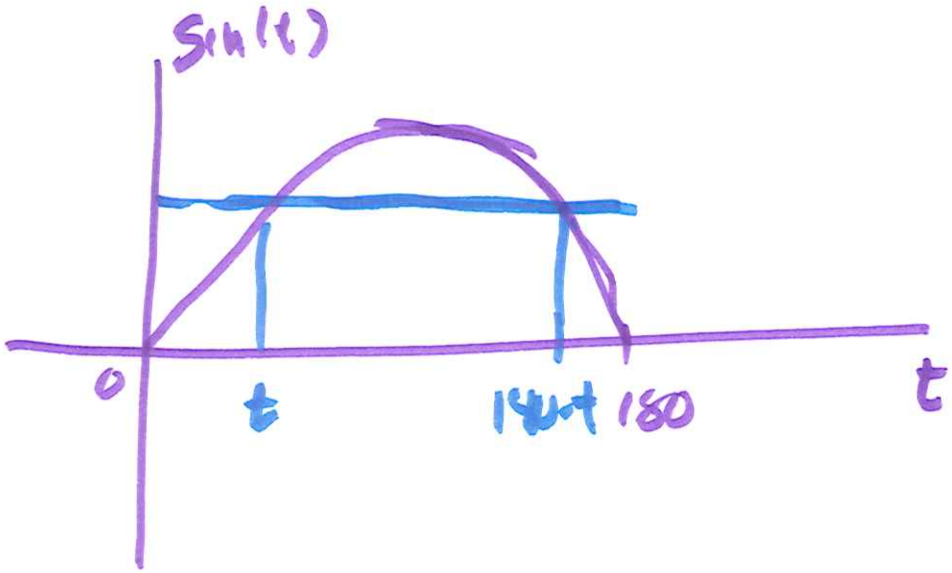
$$= \frac{10 \cdot \sqrt{3}/2}{1/2}$$

$$= \underline{10 \cdot \sqrt{3}}$$

How many solutions does the equation

$\sin(t) = 0.7$

have for t between 0 to $180°$?



$\sin(t) = c$ has

2 solutions in $[0, 180)$

if $0 \leq c < 1$.

1 solution $(t = 90)$

if $c = 1$.

0 solutions else.

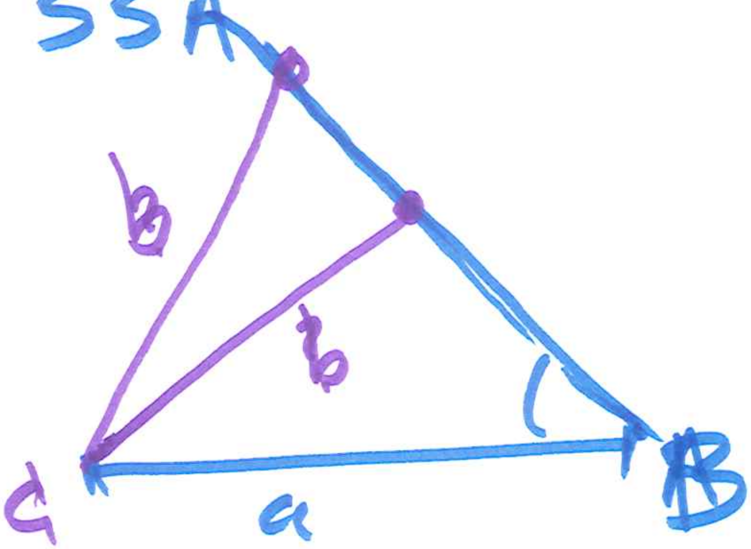
One solution is

$t = \sin^{-1}(c)$

2^{nd} solution is

$180 - \sin^{-1}(c)$

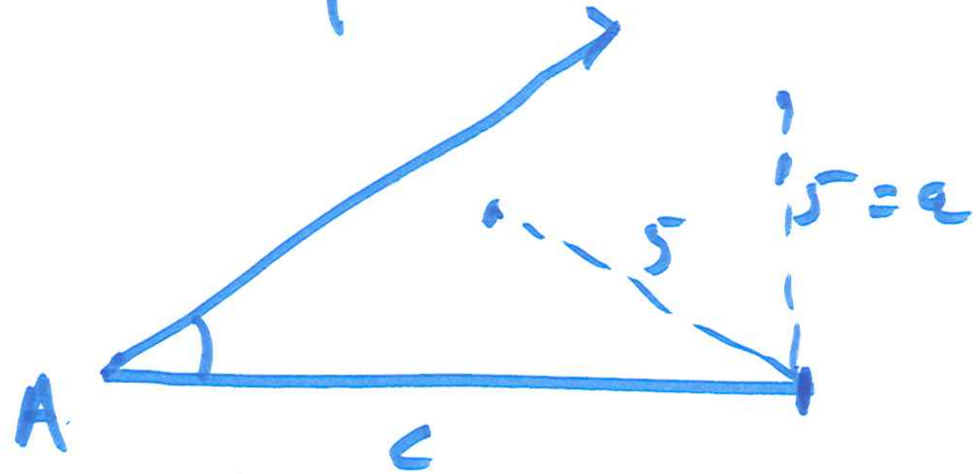
SSA



May be 2 choices for the ~~3rd side~~ angle C.

Example. Suppose $A = 45^\circ$, $c = 11$, $a = 5$. Find C.

~~Law of Sines~~.



$$\frac{\sin(A)}{a} = \frac{\sin(C)}{c}.$$

~~11 sin(A)~~

$$\frac{\sin(45)}{5} = \frac{\sin(C)}{11}$$

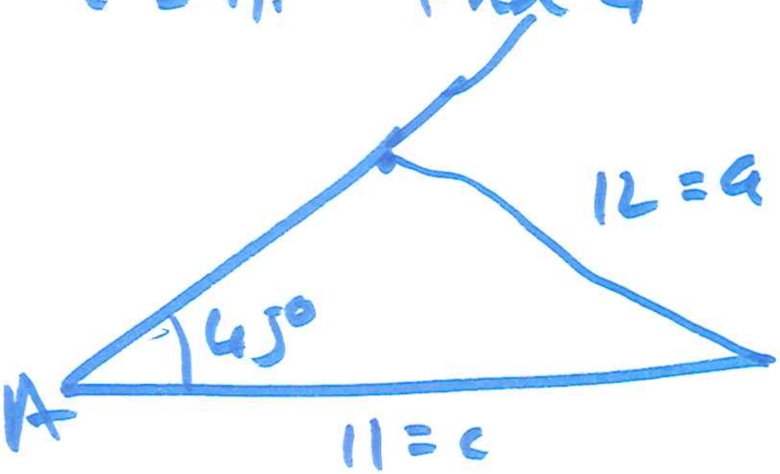
$$\sin(C) = \frac{11}{5} \cdot \sin(45)$$

$$\sin(C) = \frac{11}{5} \cdot \frac{\sqrt{2}}{2}$$

$$\approx 1.56$$

No solution.

$A = 45°$, $a = 12$, $c = 11$. Find C



$$\frac{\sin(45°)}{12} = \frac{\sin(C)}{11}$$

$$\sin(C) = \frac{11}{12} \cdot \sin(45°).$$

$$= \frac{11}{12} \cdot \frac{\sqrt{2}}{2}$$

$$\approx 0.64818.$$

$$C = \sin^{-1}(0.64818).$$

$$\approx 40.41°$$

$$\text{or } C = 180 - 40.41°$$

$$= 139.60°.$$

Discard $C' = 139.60°$

Since $A + C'$

$139.60° + 45 > 180$

Solutions

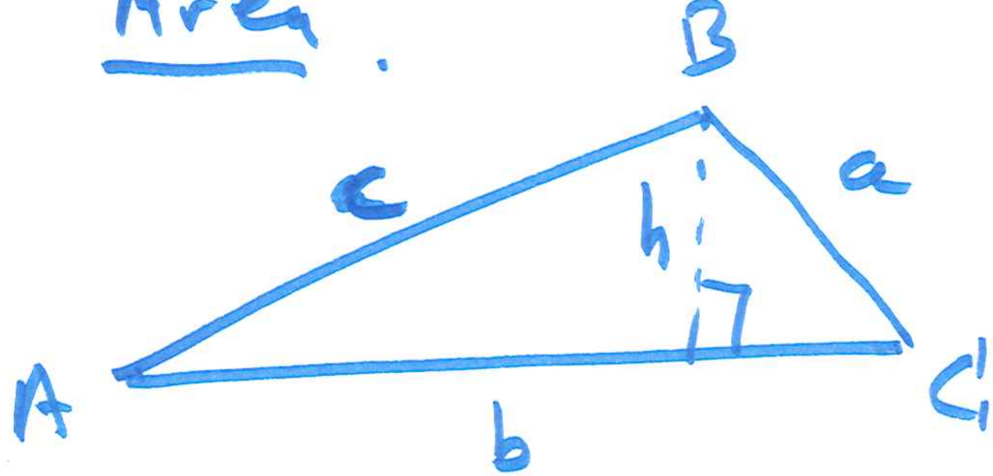
$C' = 40.41°$, $A = 45°$

$c = 11$, $a = 12$

$B = 180 - (A + C')$

b from Law of Sines.

Area



$$\text{Area} = \frac{1}{2} b \cdot h.$$

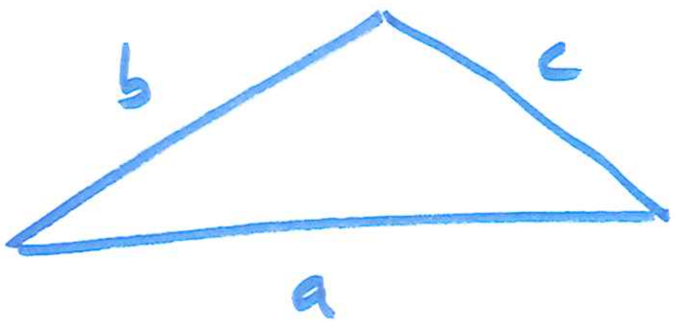
But $h = a\sin(C')$

$= c\sin(A)$

$$\text{Area} = \frac{1}{2} b a \sin(C')$$

$$= \frac{1}{2} bc \sin(A).$$

$$= \frac{1}{2} ac \sin(B).$$



$$s = \frac{1}{2}(a+b+c)$$

Area =

$$\sqrt{s(s-a)(s-b)(s-c)}$$

Heron's formula.