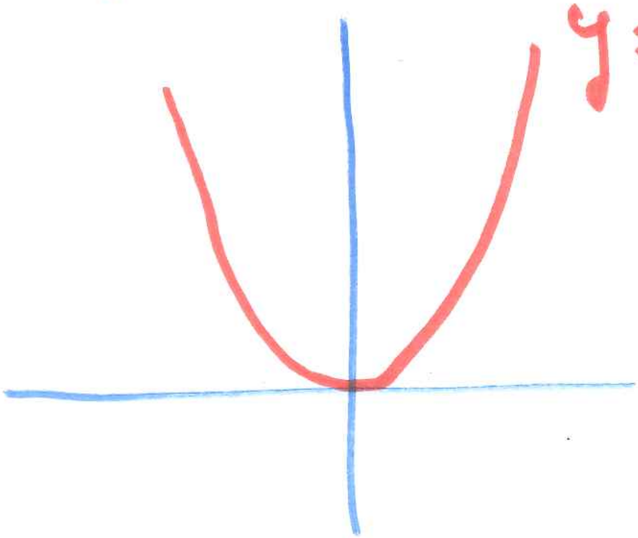
MA 110. 11/30.

- Teaching evals.
- REEF?
- Homework?

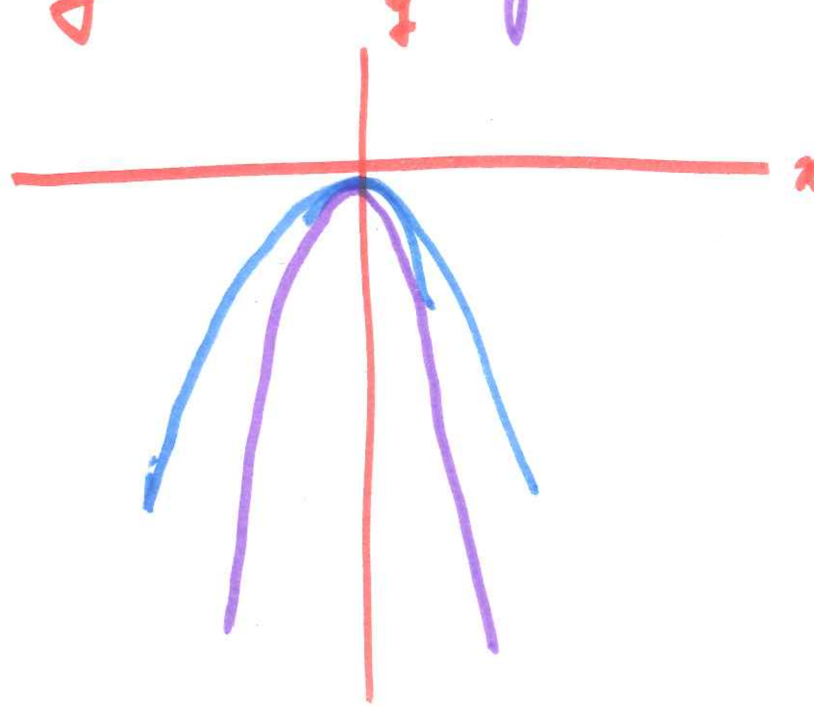
Parabolas.

$$y = x^2$$



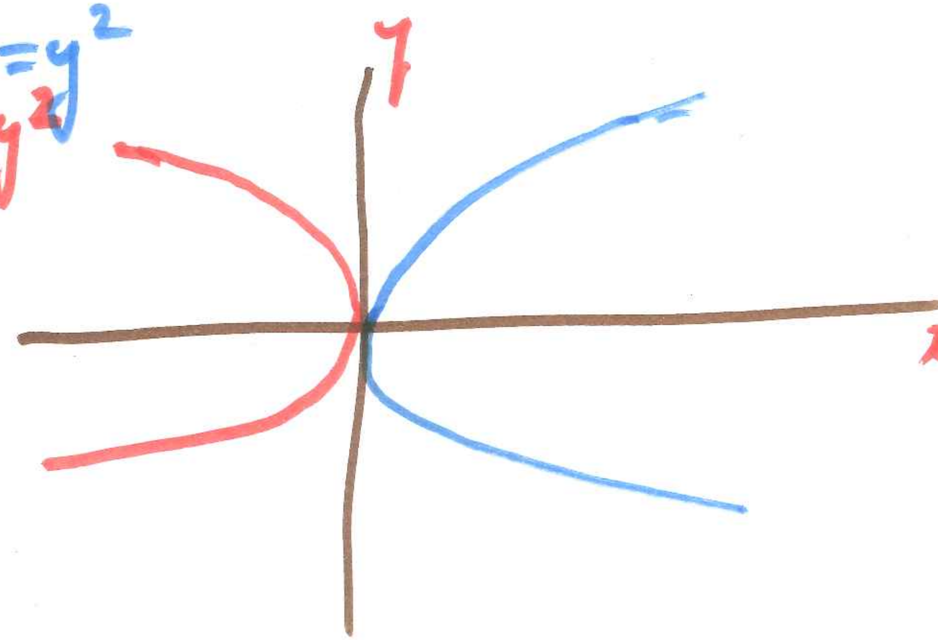
$y = x^2$

$y = -x^2, \quad y = -2x^2$

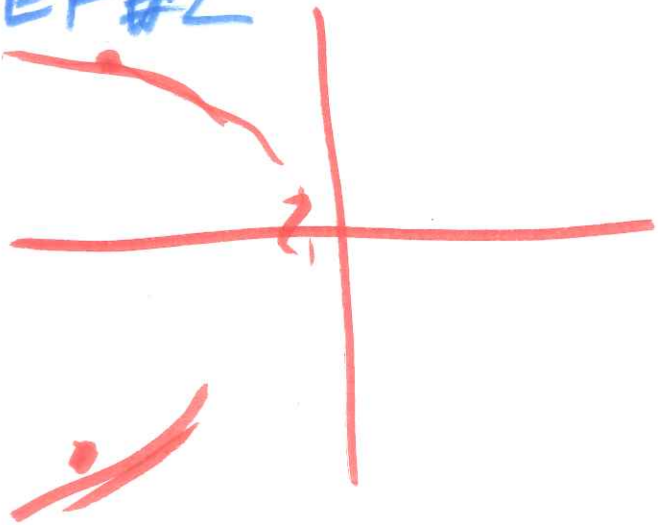


$x = y^2$

$x = -y^2$



KEEF#2

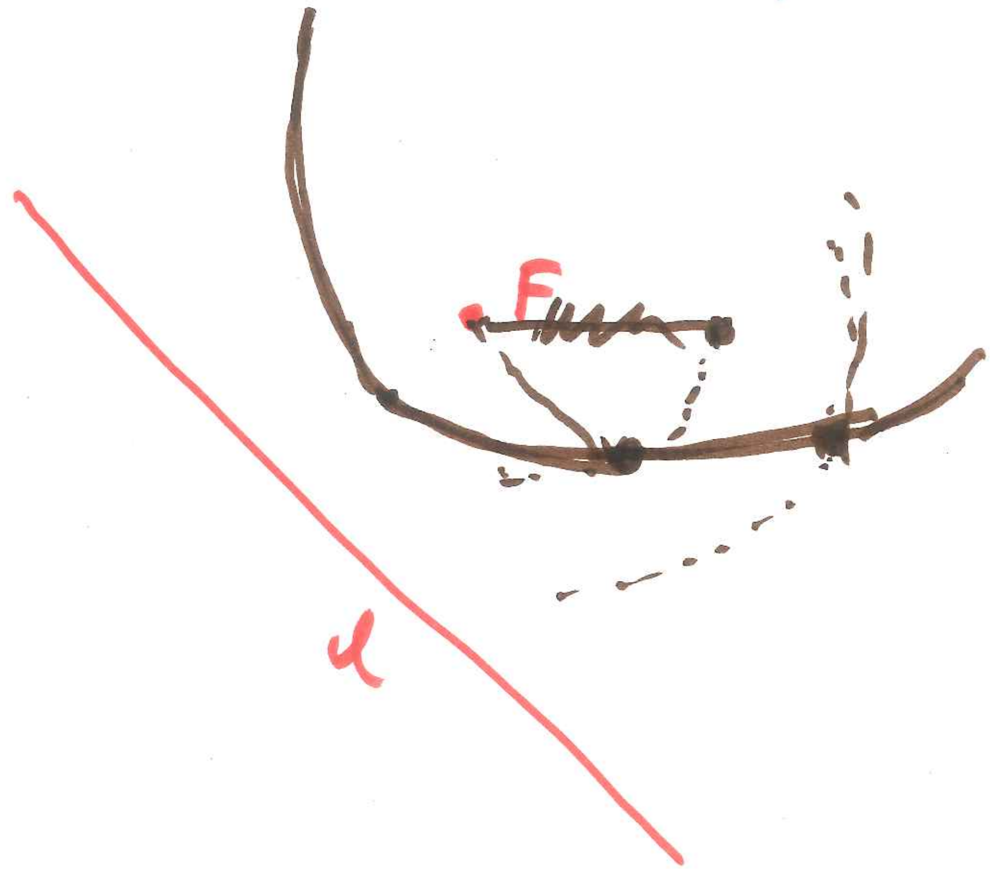


Geometric def. of a parabola –

Take a point F and a line ℓ. ~~Th~~ F is called a focus, ℓ the directrix.

Consider the set of all points P for which

$$\text{dist}(P, F) = \text{dist}(P, \ell)$$



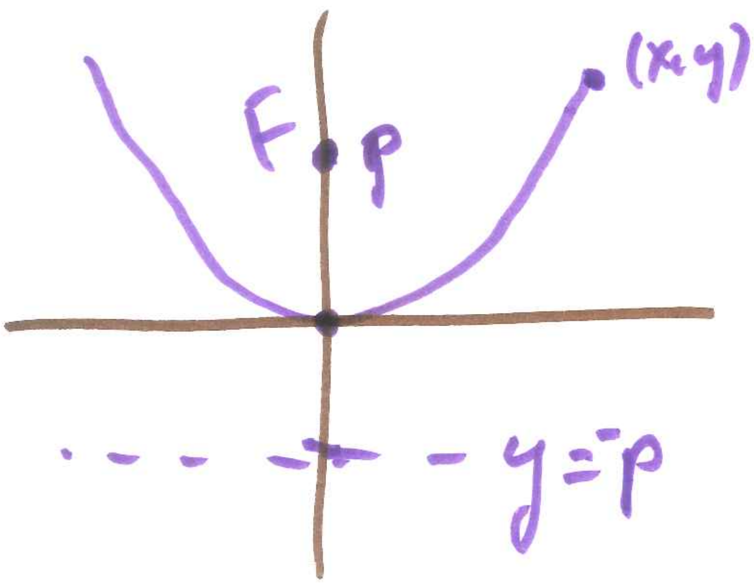
Algebraically -

Let $P = (x,y)$.

Let $F = \cancel{(p,0)}\ (0,p)$

ℓ is $y = -p$.

Here p is some given #



$\text{dist}((x,y),(p,0))$

$= \sqrt{(x-p)^2 + y^2}$

$\text{dist}((x,y), \ell)$ is $|y+p|$.

Want

$$\left(\sqrt{(x-p)^2+y^2}\right)^2 = \left(|y+p|.\right)^2$$

$$(x-p)^2 + y^2 = (y+p)^2$$

$$x^2 - 2px + p^2 + y^2 = y^2 + 2py$$

$\operatorname{dist}((x,y),(0,p))$

$= \sqrt{x^2 + (y-p)^2}$

$= (|y+p|)^2$

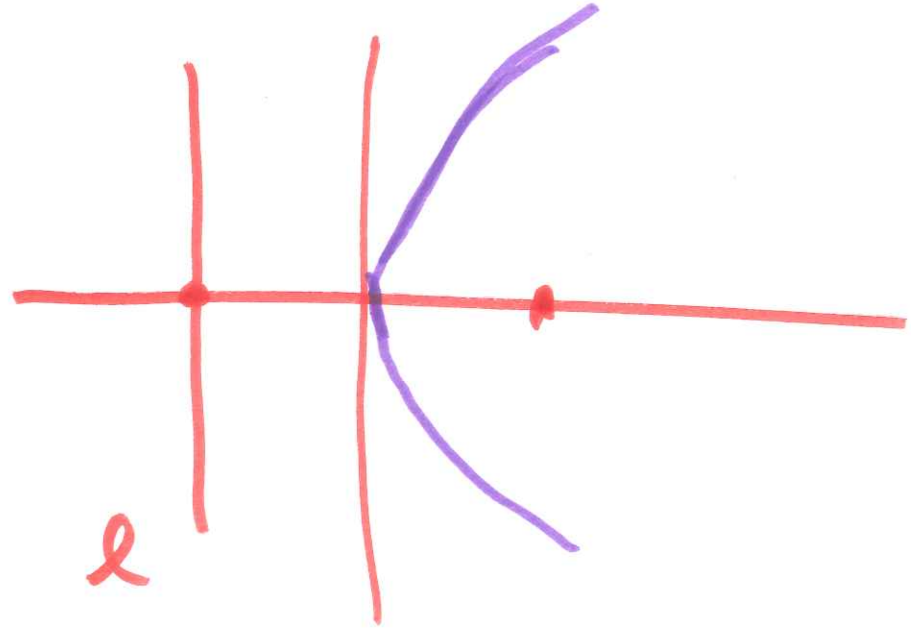
$x^2 + \cancel{y^2} - 2py + \cancel{p^2}$

$= \cancel{y^2} + 2py + \cancel{p^2}$

$x^2 = 4py.$

If $\ell : x = -p$,

$F = (p,0)$



Has equation $y^2 = 4px$

Consider

$$y = 3x^2 - 6x$$

Find vertex, focus & directrix.

$$y = 3x^2 - 6x$$

$$= 3(x^2 - 2x + 1 - 1)$$

$$\cancel{= 3((x+1)^2) -}$$

$$= 3(x-1)^2 - 3$$

$$y - 3 = 3(x-1)^2$$

$$\frac{1}{3}(y-3) = (x-1)^2.$$

Vertex is $(1,3)$.

$$\frac{1}{3}(y-3) = (x-1)^2$$

Vertex is $(1,3)$.

Shift $(0, 1/12)$ by $(1,3)$ to find Focus is $\left(1, \frac{37}{12}\right)$.

Directrix is

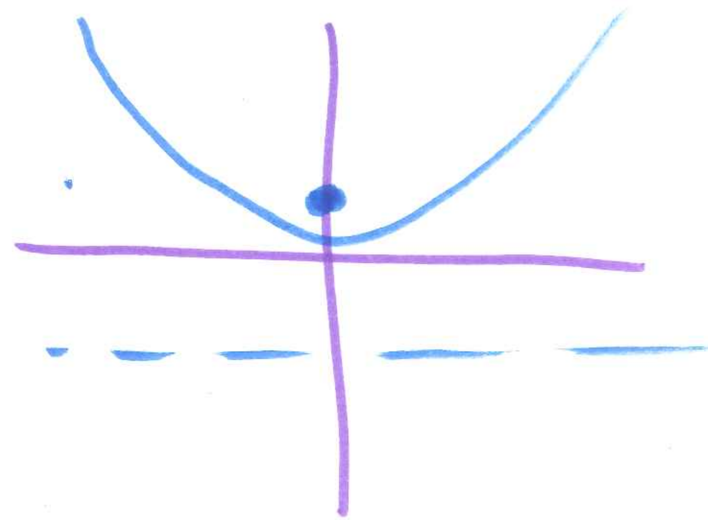
$$y = 3 - \frac{1}{12} = \frac{35}{12}.$$

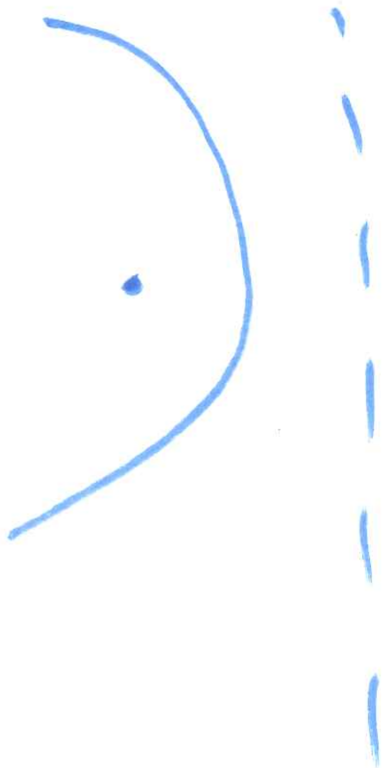
$$\frac{1}{3}y = x^2$$

~~Fo~~ $4p = \frac{1}{3}$, $p = \frac{1}{12}$.

Focus is $(0, 1/12)$.

Directrix $y = -1/12$.





$$x = -4y^2 + 4.$$

$$x - 4 = -4y^2$$

$$-\frac{1}{4}(x-4) = y^2$$

$4p = -1/4$, or $p = -1/16$

Vertex is $(4,0)$.

Graph of $-\frac{1}{4}(x-4) = y^2$ is obtained by translating $-\frac{1}{4}x = y^2$ 4 units to right.

$-\frac{1}{4}x = y^2$ has

Focus at $(-\frac{1}{16}, 0)$

Directrix at ~~y~~ $x = +\frac{1}{16}$

Then $-\frac{1}{4}(x-4) = y^2$

has Focus at

$(4 - \frac{1}{16}, 0) = (\frac{63}{16}, 0)$

Directrix is

$x = 4 + \frac{1}{16} = \frac{65}{16}$.