L 41- 12/2/16.

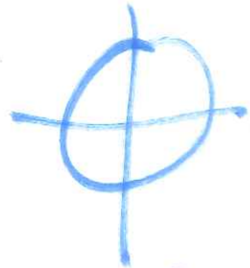
Monday 12 Dec.
Final.
- Review old tests
- HW42 (Review).

Describing ~~plane curves~~ circles.

1. $x^2+y^2=1$. The unit circle is the collection of points (x,y) s.t.

$$x^2+y^2=1$$

- Implicit representation.



2. Graph of functions

$$y=\pm\sqrt{1-x^2}$$

3. Parametric curve.

$$(\cos(t), \sin(t)), \quad 0\le t\le 2\pi.$$

Plane curve.

If f, g are continuous functions.. then ~~the~~ we may define a curve by $x=f(t)$, $y=g(t)$

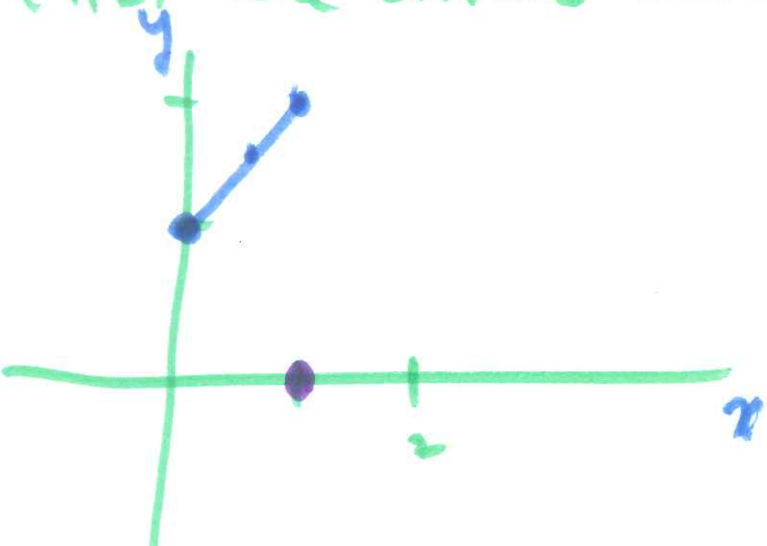
$a\le t\le b$.

Example.

$x(t) = t, \ y(t) = t+1$

t	x	y
0	0	1
$1/2$	$1/2$	$3/2$
1	1	2

The points $(0,1)$, $(\frac{1}{2}, 3/2)$, $(1,2)$ are on this curve



Example

$x(t) = t+1, \ y(t) = t^2+3t.$

Find an equation in x, y which contains the curve.

Eliminating the parameter t.

$x = t+1, \ y = t^2+3t.$

Choose one equation to solve for t. (Pick the easier one).

$t = \underline{x-1}$

Substitute the expression for t into the other equation

$$y = t^2 + 3t$$

$$= (x-1)^2 + 3(x-1)$$

$$= x^2 - 2x + 1 + 3x - 3$$

$$= x^2 + x - 2.$$

REEF #2.

Eliminate t to find an equation whose graph contains the curve

$$(x,y) = (2t+1,\ t^2).$$

<u>Solution.</u>

$2t+1 = x$, ~~$x = \frac{1}{2}$~~

$$t = \frac{1}{2}(x-1)$$

$$y = t^2 = \left(\frac{1}{2}(x-1)\right)^2$$

$$= \frac{1}{4}(x^2 - 2x + 1).$$

Find a parametric description of the line through $(1,2)$ with slope -2.

Find another -

Solution 1 Use point slope

$$y-2 = -2(x-1)$$

$$y = -2x+2+2$$

$$= -2x+4.$$

Let $x=t$

$(t, -2t+4)$.

Solution 2

$x = 1+t$, $y = 2-2t$.

$$\text{Slope} = \frac{\text{change in } y}{\text{change in } x}$$

Solution 3

Try $x = 42t$.

$$y = -2x+4$$

$$= -2(42t)+4$$

$$= -84t+4$$

Circles.

$x = \cos(t)$, $y = \sin(t)$

describes the unit circle w/ center O.

$x = a\cos(t)$,

$y = a\sin(t)$

Describes a circle of radius $a > 0$.

$x = h + a\cos(t)$

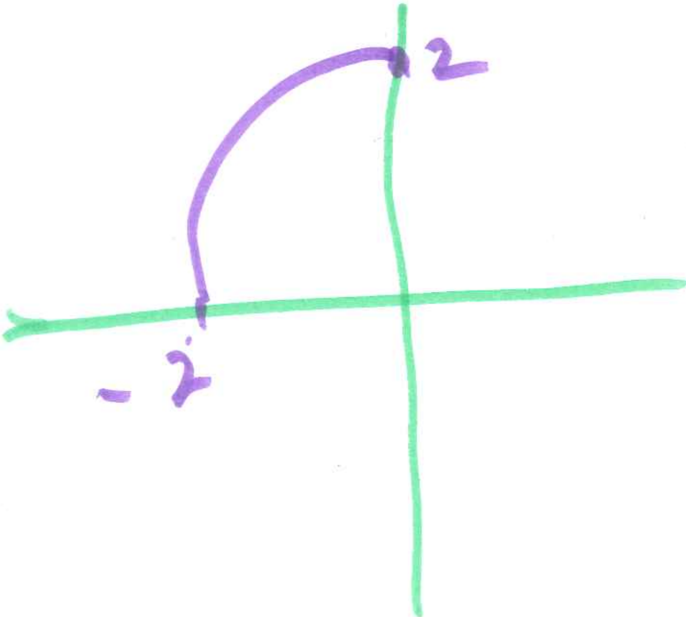
$y = k + a\sin(t)$

has center (h, k) radius $a > 0$.

Example. Find a parametric description of the part of the circle of radius 2 center O that lies in the 2^{nd} quadrant.

Find a parametric rep. of the top half of the ellipse

$$\frac{x^2}{4} + \frac{y^2}{9} = 1.$$



$x = 2\cos(t)$

$y = 2\sin(t)$

$\frac{\pi}{2} \le t \le \pi.$

$x = 2\cos(t)$, $y = 3\sin(t)$

$$\frac{(2\cos(t))^2}{4} + \frac{(3\sin(t))^2}{9} = \cos^2(t) + \sin^2(t) = 1.$$

$0 \le t \le \pi$, given $x \ge 0$.

A batted ball travels along the curve

$x(t) = 20t$

$y(t) = 1 + 30t - 5t^2$

will it make it over a fence 110 m from ~~the~~ $x=0$ to 3 ~~feet~~ m high.

$x = 110$

$20t = 110$ or $t = 5.5$ s.

$y = 1 + 30 \cdot 5.5 - 5(5.5)^2$

$\cancel{118} = 14.75 > 3.$

Yes.