

MA110. 12/5 - #42

Review -

Example 1. $\text{If } \pi \leq t \leq \frac{3\pi}{2}$

$\sin(t) = -5/13$. Find

$\sec(t)$.

$$\cos^2(t) + \sin^2(t) = 1.$$

Since $(\cos(t), \sin(t))$
are on unit circle.

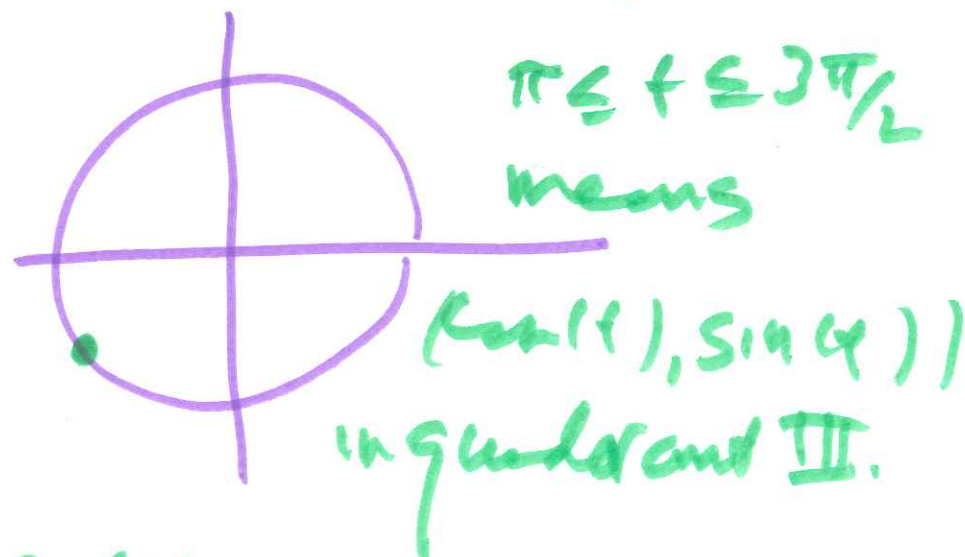
$$\cos^2(t) + \left(-\frac{5}{13}\right)^2 = 1.$$

$$\cos^2(t) = 1 - \frac{25}{169}$$

$$= \frac{169 - 25}{169}$$

$$= \frac{144}{169}.$$

$$\cos(t) = \pm 12/13.$$



$$\cos(t) < 0, \sin(t) < 0$$

$$\text{Choose neg. sign } \cos(t) = -\frac{12}{13}.$$

Since $\sec(t) = \frac{1}{\cos(t)}$
 $\sec(t) = \frac{1}{(-12/13)} = -\frac{13}{12}$.

Example 2

Find $\sin^{-1}(\sin(2\pi))$.
 $= 0$

$t = \sin^{-1}(\sin(2\pi))$ is
the angle in $[-\frac{\pi}{2}, \frac{\pi}{2}]$
which satisfies

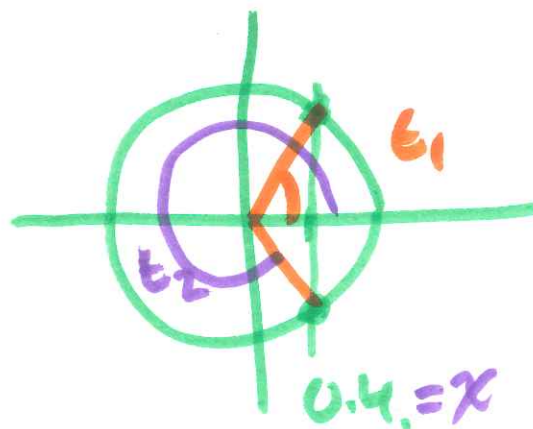
$$\sin(t) = \sin(2\pi)$$

$$= 0$$

or $t = 0$.

Example 3.

Find all solutions
of $\cos(t) = 0.4$.
in $[0, 2\pi]$.

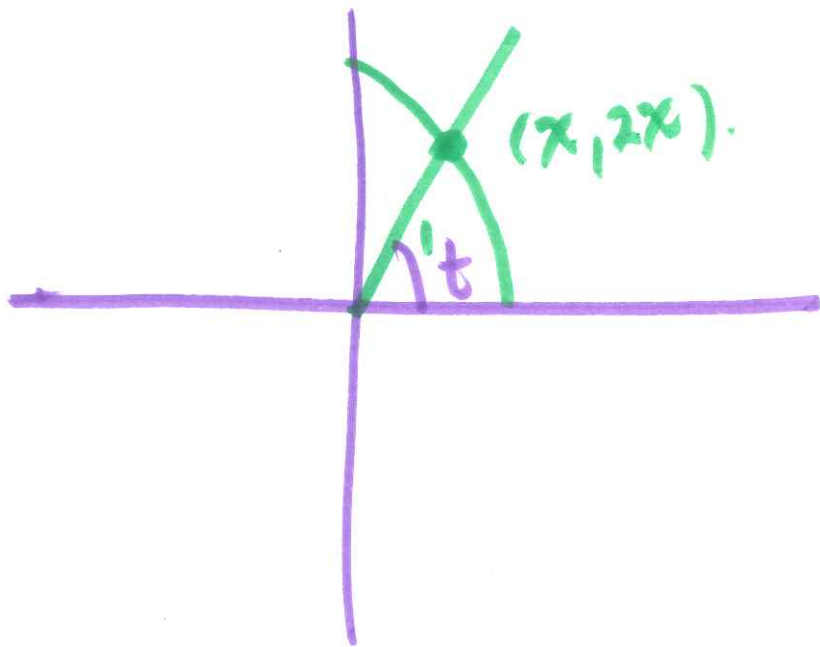


$$t_1 = \cos^{-1}(0.4)$$

$$\approx 1.159.$$

$$t_2 = 2\pi - t_1 \approx 5.1239.$$

Round 2 dec. places 1.16
and 5.12.



Find the angle between
the ray $\{y=2x, x>0\}$
and the positive x-axis

$$x^2 + (2x)^2 = 1. \quad x = \frac{1}{\sqrt{5}}$$

$$5x^2 = 1$$

$$x^2 = \frac{1}{5}$$

$$y = 2x = \frac{2}{\sqrt{5}}$$

$$\sin(t) = \frac{y}{r} = \frac{2/\sqrt{5}}{5}$$

$$\cos(t) = \frac{x}{r} = \frac{1/\sqrt{5}}{5}$$

$$\tan(t) = \frac{y}{x} = \frac{2/\sqrt{5}}{1/\sqrt{5}} = 2$$

$$t = \sin^{-1}\left(\frac{2}{5}\right)$$

$$t = \cos^{-1}\left(\frac{1}{5}\right)$$

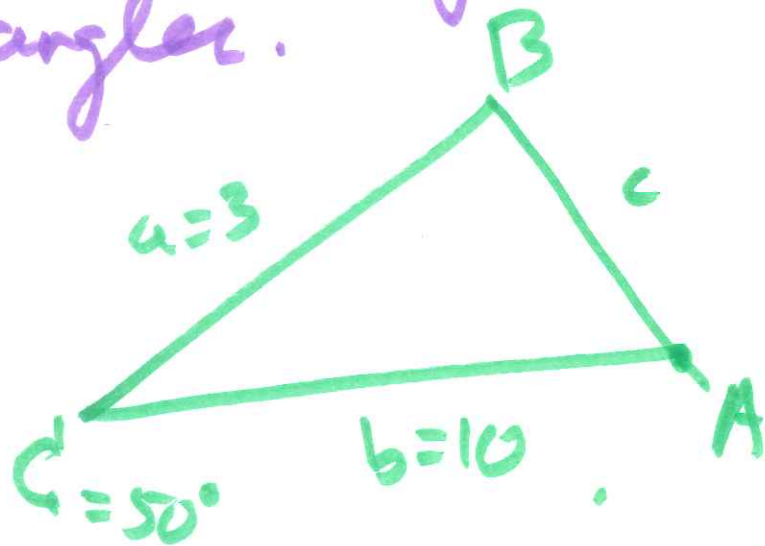
$$t = \tan^{-1}(2).$$

For a triangle

$$C = 50^\circ, a = 3$$

$$b = 10$$

Find remaining sides & angles.



SAS - Use Law of Cosines - to find 3rd side.

$$c^2 = a^2 + b^2 - 2ab \cos(C)$$
$$= 3^2 + 10^2 - 2 \cdot 3 \cdot 10 \cdot \cos(50)$$

$$\approx 70.4327 \dots$$

$$c \approx 8.3924 \dots$$

Find A. Law of Cosines again.

$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

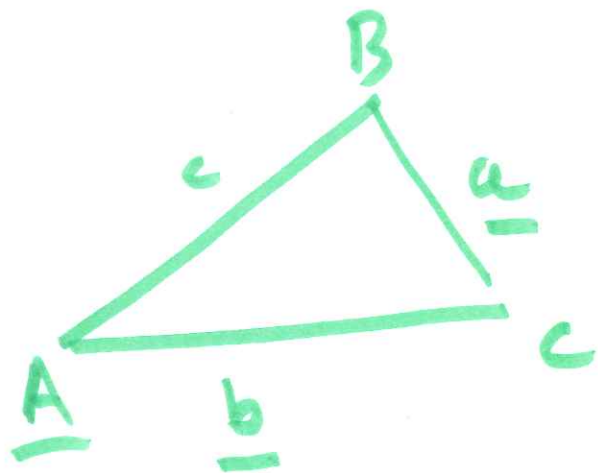
$$\cos(A) = \frac{c^2 + b^2 - a^2}{2bc}$$

$$A = \cos^{-1}\left(\frac{c^2 + b^2 - a^2}{2bc}\right)$$

$$\approx 15.8925^\circ$$

$$B = 180 - (A + C)$$

$$\approx 134.1074$$

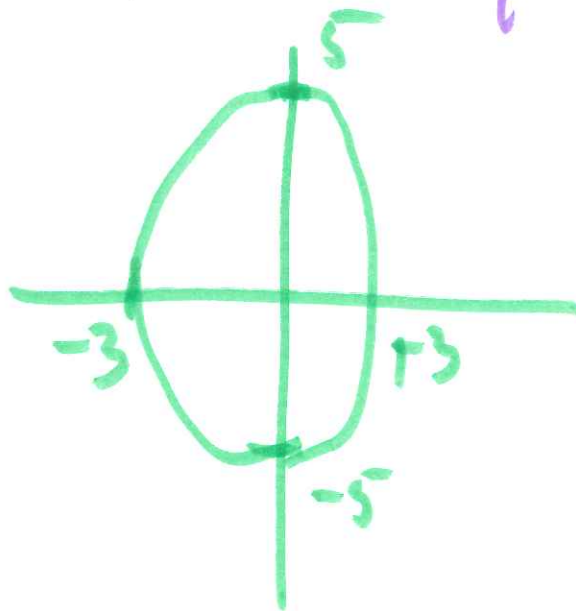


SSA. Use Law of Sines

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b}$$

The major axis ellipse is on the y-axis & of length 10.

The minor axis is of length 6. Center (0,0)
Find the equation -



Equation is

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1.$$

$$0 + \frac{5^2}{a^2} = 1$$

$$a = 5$$

$$b = 3$$

$$\frac{x^2}{9} + \frac{y^2}{25} = 1.$$

Sketch ellipse given by

$$25x^2 + 50x + 9y^2 - 72y = 56$$

$$25(x^2 + 2x + 1)$$

$$+ 9(y^2 - 8y + 16) = 56$$

$$+ 25$$

$$+ 144$$

$$144$$

$$25(x+1)^2 + 9(y-4)^2 = 225.$$

$$\frac{25(x+1)^2}{225} + \frac{9(y-4)^2}{225} = 1.$$

$$\frac{(x+1)^2}{9} + \frac{(y-4)^2}{25} = 1.$$

$$\frac{(x+1)^2}{9} + \frac{(y-4)^2}{25} = 1.$$

Ellipse $\frac{x^2}{9} + \frac{y^2}{25} = 1$

shifted 1 unit to
the left & 4 units
up