Lecture 29: More graphs of trig functions

Russell Brown

Department of Mathematics
University of Kentucky

Question 1.

Find the period of the function $f(t) = \sin(\pi t/12)$.

A 12

B 12π

C $24/\pi$

D 24

E $12/\pi$

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The period of $\sin(t)$ is 2π. Thus, $\sin(\pi t/12)$ will complete one period if $0 \leq \pi t/12 \leq 2\pi$. Solving this pair of inequalities, gives $0 \leq t \leq 24$. Thus, the period is 24.

Question 2.

List the transformations needed to transform the graph of $\sin(x)$ into the graph $\sin(3x + \pi)$. There is more than one correct answer, but you only need to find one.

- A Shift π units to the left and then shrink horizontally by factor of 3.
- B Shrink horizontally by a factor of 3 and then shift $\pi/3$ units to the left.
- C Shrink horizontally by a factor of 3 and then shift π units to the left.
- D Shrink horizontally by a factor of 3 and then shift $\pi/3$ units to the right.
- E Shrink horizontally by a factor of 3 and then shift π units to the right.

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Solution to 2.

If we shift π units to the left we transform the graph to $\sin(x+\pi)$. Then shrinking horizontally by a factor of 3 gives $\sin(3x+\pi)$. Thus A is correct.

If we shrink horizontally by a factor of three, we obtain the graph of $\sin(3x)$ and then translating left by $\pi/3$ gives $\sin(3(x+\pi/3)) = \sin(3x+\pi)$. Thus B is correct.

If we shrink horizontally by a factor of three, and then translate π units to the left, we obtain $\sin(3x+3\pi)$. Since sin is periodic with period π, we have $\sin(3x+3\pi) = \sin(3x+\pi)$ and C is also a correct answer.

If we shrink horizontally by a factor of 3 and then shift $\pi/3$ units to the right gives $\sin(3(x-\pi/3)) = \sin(3x-\pi)$. Since sin is periodic with period π, we have $\sin(3x-\pi) = \sin(3x+\pi)$. Thus D is correct.

If we shrink horizontally by a factor of 3 and then shift π units to the right gives $\sin(3(x-\pi)) = \sin(3x-3\pi)$. Since sin is periodic with period π, we have $\sin(3x-3\pi) = \sin(3x+\pi)$. Thus E is correct.

In fact, all five answer are correct!

Question 3.

What shift should we apply to the graph of $\sin(x)$ to obtain the graph of $\cos(x)$? Again, there are multiple right answers.

- A π units to the right
- B $\pi/2$ units to the left
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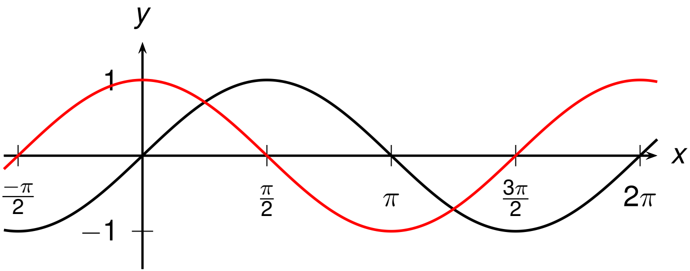
Question 3–solution

The black curve is $\sin(x)$ and the red curve is $\cos(x)$. We may shift the black curve $\pi/2$ units to the left or $3\pi/2$ units to the right to obtain the red curve.



Question 4.

The terminal side of an angle t is the part of the line $y = 3x$ with $x \leq 0$. Find $\cos(t)$, $\sin(t)$ and $\tan(t)$. Hint: Find a point on the line $y = 3x$ with $x < 0$.

- A $\cos(t) = -3\sqrt{10}/10$, $\sin(t) = -\sqrt{10}/10$, $\tan(t) = 3$.
- B $\cos(t) = -\sqrt{10}/10$, $\sin(t) = 3\sqrt{10}/10$, $\tan(t) = 1/3$.
- C $\cos(t) = 3\sqrt{10}/10$, $\sin(t) = \sqrt{10}/10$, $\tan(t) = 3$.
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We know that $x < 0$ so we choose a negative number for x. A convenient choice is $x = -1$. Then $y = 3 \cdot (-1) = -3$ so that $(-1, -3)$ is a convenient point on the line. This point is $\sqrt{1^2 + 3^2} = \sqrt{10}$ units from the origin. Thus a point on the line $y = 3x$ and on the unit circle is $(-1/\sqrt{10}, -3/\sqrt{10}) = (-\sqrt{10}/10, -3\sqrt{10}/10)$.
From this we have $\cos(t) = -\sqrt{10}/10$, $\sin(t) = -3\sqrt{10}/10$ and $\tan(t) = \sin(t)/\cos(t) = 3$.
Unfortunately, none of the options in the question are correct. Full credit was given to everyone on this question.