Lecture 30: Other trigonometric functions

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Question 1.

(There was a typo in the question on Monday. Here is a corrected version.)

An angle of radian measure t is in standard position and the terminal side is the part of the line $y = 3x$ with $x \leq 0$. Find $\cos(t)$, $\sin(t)$ and $\tan(t)$. Hint: Find a point on the line $y = 3x$ with $x < 0$.

- A $\cos(t) = -3\sqrt{10}/10$, $\sin(t) = -\sqrt{10}/10$, $\tan(t) = 3$.
- B $\cos(t) = -\sqrt{10}/10$, $\sin(t) = 3\sqrt{10}/10$, $\tan(t) = 1/3$.
- C $\cos(t) = 3\sqrt{10}/10$, $\sin(t) = \sqrt{10}/10$, $\tan(t) = 3$.
- D $\cos(t) = -3\sqrt{10}/10$, $\sin(t) = \sqrt{10}/10$, $\tan(t) = -3$.
- E $\cos(t) = -\sqrt{10}/10$, $\sin(t) = -3\sqrt{10}/10$, $\tan(t) = 3$.

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We know that $x < 0$ so we choose a negative number for x, say $x = -1$. Then $y = 3 \cdot (-1) = -3$ so that $(-1, -3)$ is on the terminal side of the angle. This point is $\sqrt{1^2 + 3^2} = \sqrt{10}$ units from the origin. Thus a point on the line $y = 3x$ and on the unit circle is $(-1/\sqrt{10}, -3/\sqrt{10}) = (-\sqrt{10}/10, -3\sqrt{10}/10)$.
From this we have $\cos(t) = -\sqrt{10}/10$, $\sin(t) = -3\sqrt{10}/10$ and $\tan(t) = \sin(t)/\cos(t) = 3$.

Question 2.

Suppose that $\cos(t) < 0$ and $\tan(t) > 0$, which of the following inequalities might be true?

A $0 < t < \pi/2$

B $\pi/2 < t < \pi$

C $\pi < t < 3\pi/2$

D $3\pi/2 < t < 2\pi$

E $-\pi/2 < t < 0$

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Since $\cos(t) < 0$ and $\tan(t) = \sin(t)/\cos(t) > 0$, we have $\sin(t) < 0$. With $\sin(t) < 0$ and $\cos(t) < 0$, we conclude we are in the third quadrant. Thus one choice is that t lies in the interval $[\pi/2, 3\pi/2]$.

Question 3.

Find the solution(s) of the equation $\tan(2x) = 1$ with $0 < x < \pi$.

A $\pi/4$, $5\pi/4$

B $\pi/8$

C $\pi/8$, $5\pi/8$

D $\pi/4$

E 42π

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The function $\tan(x)$ is periodic with period π. Thus $\tan(2x)$ has period $\pi/2$. We need to consider two periods of the graph. From the unit circle $\tan(t) = 1$ if $t = \pi/4$ and $5\pi/4$. Thus, we want $2x = \pi/4, 5\pi/4$ or $x = \pi/8, 5\pi/8$.

Question 4.

Simplify the expression by writing everything in terms of $\sin(x)$ and $\cos(x)$.

$$\tan(x)\cot(x)\cos(x)(1+\sec^2(x))$$

A $\cos(x)$

B $\tan^2(x)$

C $\cos(x) + 1/\cos(x)$

D 1

E None of the above.

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D 1

E None of the above.

If we write everything in terms of $\sin(x)$ and $\cos(x)$, we obtain

$$\begin{aligned} &\tan(x)\cot(x)\cos(x)(1+\sec^2(x)) \\ =\; & \frac{\sin(x)}{\cos(x)}\frac{\cos(x)}{\sin(x)}\cos(x) + \frac{\cos(x)}{\cos^2(x)} \\ =\; & \cos(x) + \frac{1}{\cos(x)} \end{aligned}$$