Lecture 35: Solving trigonometric equations

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Question 1.

Solve the equation $\cos(x) = 1$ for x. There may be more than one right answer below.

A 0

B 2π

C 4π

D 6π

E 42π

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For any integer k, we have $\cos(2k\pi) = \cos(0) = 1$. Thus all answers are correct.

We will typically write this as $x = 2k\pi$ $(k = 0, \pm 1, \pm 2, \dots)$.

Question 2.

Solve $\tan(x) = 1$. Give all solutions.

A $\tan^{-1}(1)$

B $\pi/4$

C $\pi/4 + 2k\pi \ (k = 0, \pm 1, \pm 2, \dots)$

D $\dfrac{\pi + 4k\pi}{4} \ (k = 0, \pm 1, \pm 2, \dots)$

E $-\pi/4 + k\pi \ (k = 0, \pm 1, \pm 2, \dots)$

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We know that $\tan(\pi/4) = 1$ and thus $\tan^{-1}(1) = \pi/4$. Since $\tan(x + k\pi) = \tan(x)$, other solutions of $\tan(x) = 1$ are of the form

$$\frac{\pi}{4} + k\pi = \frac{\pi}{4} + \frac{4k\pi}{4} = \frac{\pi + 4k\pi}{4}$$

for $k = 0, \pm 1, \pm 2, \pm 3, \dots$.

Question 3.

Find all solutions of $\cos(x) = 2$. n

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B $\pi/4 + k\pi$, $k = 0, \pm 1, \pm 2, \ldots$

C $\pi/4$

D $\pi/4 + 2k\pi$ and $-\pi/4 + 2k\pi$, $k = 0, \pm 1, \pm 2, \ldots$

E None of the above.

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E None of the above.

E.

Since the range of cos is $[-1, 1]$, there is no solution to the equation $\cos(x) = 2$.

Question 4.

Solve $\sec(x) = 2$. Give all solutions.

- A $\pi/3 + 2k\pi$, $k = 0, \pm 1, \pm 2, \ldots$
- B $\pi/3 + 2k\pi$ and $-\pi/3 + 2k\pi$, $k = 0, \pm 1, \pm 2, \ldots$
- C $\pi/3 + 2k\pi$ and $2\pi/3 + 2k\pi$, $k = 0, \pm 1, \pm 2, \ldots$
- D $\pi/3 + k\pi$, $k = 0, \pm 1, \pm 2, \ldots$
- E $-\pi/3 + k\pi$, $k = 0, \pm 1, \pm 2, \ldots$

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Recall that $\sec(x) = 1/\cos(x)$, thus if $\sec(x) = 2$, then $\cos(x) = 1/2$. The solutions of $\cos(x) = 1/2$ are in option B.