1. Carry out the following steps to sketch the graph of

$$f(x) = \frac{x}{1 + x^2}.$$

   (a) Find the local maxima and minima for f. Compute the local maximum and minimum values. Give each intervals of increase and decrease.

   (b) Find the inflection points for f. Give the intervals where f is increasing and decreasing.

   (c) Determine if f is even or odd.

   (d) Find $\lim_{x \to \infty} f(x)$ and $\lim_{x \to -\infty} f(x)$.

   (e) Draw a graph which reflects the above information.

2. Suppose that we are constructing a cylindrical can with base a circle of radius r and height h. Assume the can has no top. The volume is to be $1000\pi\text{cm}^3$. Carry out the following steps to find the dimensions of the can with least surface area.

   (a) Write a function which gives the surface area as a function of the radius r. For what values of r is your function defined?

   (b) Find the absolute minimum value of the function you found in part a).

   (c) Explain briefly how you may use calculus to show the value in part b) is a minimum.

   (d) Give the dimensions of the can of smallest surface area.

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