

w. 8/26/15.

①

Functions

$$f(x) = 2x + 1. \quad f(2x) = 4x + 1$$

$$f(3) = 7$$

A function $f: D \rightarrow Y$
is a rule that associates
a real $f(x)$ in Y . to each
 x in the set D

D is the domain

Y is the range

~~Not a~~

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Natural domain
for a function is the
largest set where the
formula is defined

Examples

$$f(x) = 2 - \sqrt{x-3}$$

$$g(x) = 2 + \frac{1}{x-1}$$

Find natural domain

Find range

f is defined if $x-3 \geq 0$

$\Rightarrow x \geq 3$. Domain is $\{x: x \geq 3\}$

$\text{or } [3, \infty) = \{x: 3 \leq x < \infty\}$

$$f(x) = 2 - \sqrt{x-3}$$

Range is $\{x: x \leq 2\}$ or $(-\infty, 2]$

$$g(x) = 2 + \frac{1}{x-1}$$

Domain Need $x-1 \neq 0$
 $\wedge x \neq 1.$

$$\{x: x \neq 1\} \wedge (-\infty, 1) \cup (1, \infty)$$

Range $g(x)$ is in the range
 if we can solve

$$y = 2 + \frac{1}{x-1}$$

$$(x-1) \frac{y-2}{y-2} = \frac{1}{x-1} \frac{(x-1)}{y-2} \quad \text{if } y-2 \neq 0$$

$$x-1 = \frac{1}{y-2} \wedge x = 1 + \frac{1}{y-2}$$

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Range is $\{y: y \neq 2\}$. 4

Composition.

If f, g are functions, the composite function is

$$f \circ g(x) = f(g(x)).$$

Find $f \circ g$ if $f(x) = x + 1$
 $g(x) = \sqrt{x}$.

$$f \circ g(x) = f(\sqrt{x}) = \sqrt{x} + 1.$$

One-one functions

We say f is 1-1 if
for each y , the equation
 $f(x) = y$ has at most one
solution.

Inverse function.

If f is a one-to-one function, then f^{-1} is function with $f^{-1}(y) = x$ when $f(x) = y$.

Example.

$$f(x) = x^2 + 2x$$

Is f one-to-one.

-No. $f(x) = 0$ has 2

Solutions, $x = 0$ and $x = -2$.

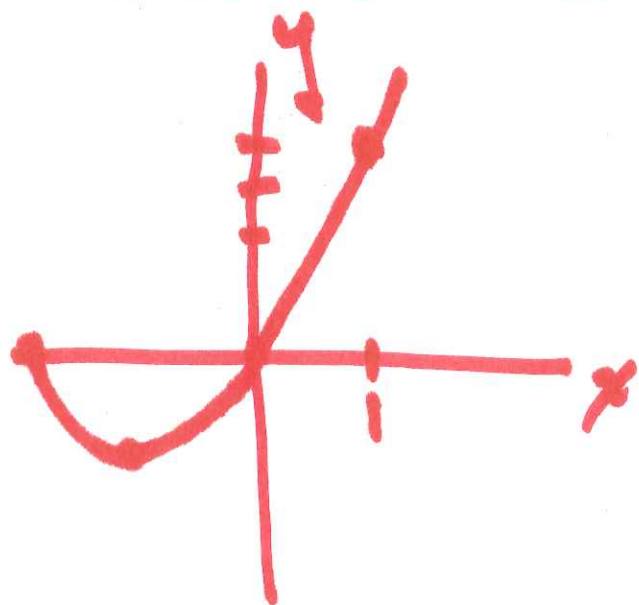
Can you choose a domain so that f is 1-1?

Graph f

$$f(x) = x^2 + 2x + 1 - 1.$$

$$= (x+1)^2 - 1.$$

Vertex is $x = -1$, $y = -1$.



f is 1-1 on

$[-1, \infty)$, $\checkmark$

$(-\infty, -1]$

$(-2, -1) \cup (7, 8)$

~~Find f^{-1}~~ Give f
the domain $[-1, \infty)$.
find f^{-1} .

Solve

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$$y = f(x).$$

$$y = (x+1)^2 - 1.$$

$$y+1 = (x+1)^2$$

$$\pm \sqrt{y+1} = x+1.$$

$$x = -1 \pm \sqrt{y+1}.$$

Choose "+"-sign so
that x is $[1, \infty)$

$$f^{-1}(y) = -1 + \sqrt{y+1}.$$

Graph f^{-1} .

Give domain & range
of f^{-1} and f

~~f domain is~~

~~f domain~~

range

f $[-1, \infty)$

$[-1, \infty)$

f^{-1} $[-1, \infty)$

$[-1, \infty)$

