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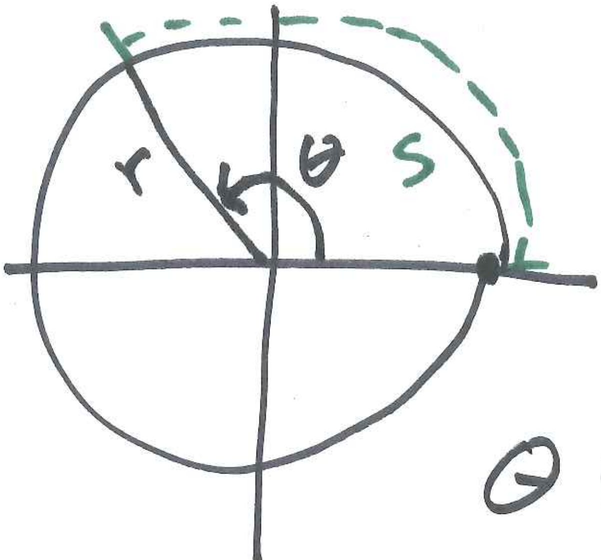
Trig functions.

Radian measure.



r = radius

s = arclength.

$$\underline{\underline{\theta = s/r}}$$

θ is the radian measure of the angle.

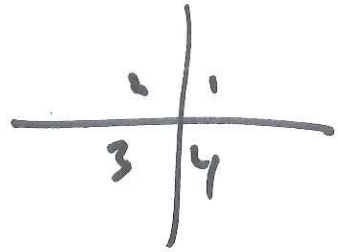
~~Trig functions.~~

<u>Example.</u> What is the radian measure of a full revolution?

Arc is $2\pi r$. radius is r

$$\theta = {}^{2\pi r}/_{r} = 2\pi.$$

A right angle?



$$2\pi/4 = \pi/2.$$

One degree?

$$\frac{2\pi}{360} = \frac{360^\circ}{360}$$

$$1^\circ = \pi/180.$$

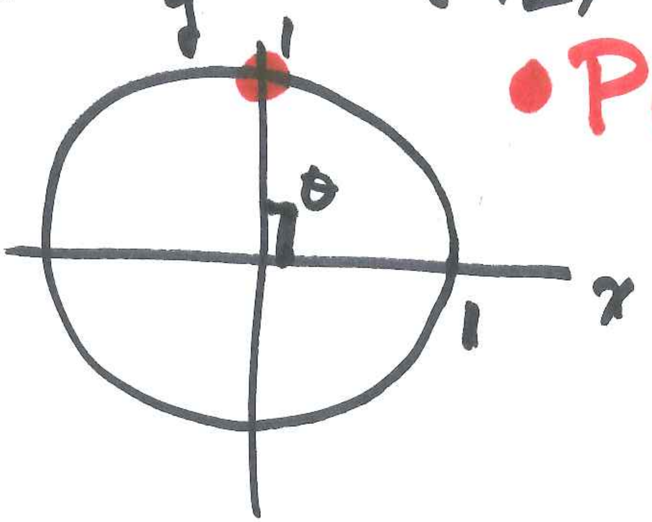
Trig Functions.

- Pick an angle θ
- Draw θ on the unit circle.
- Call the point where the ray of angle θ meets the circle $P(\theta)$

③ Then

$P(\theta) = (\cos(\theta), \sin(\theta))$.

Find $\cos(\pi/2)$, $\sin(\pi/2)$.



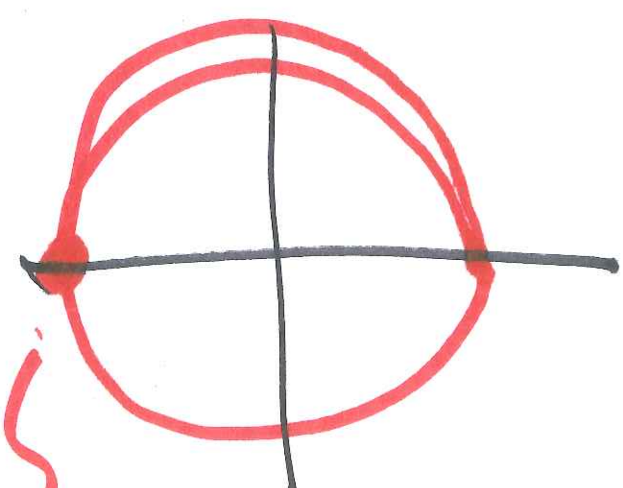
• $P(\theta) = (0, 1)$.

$\cos(\frac{\pi}{2}) = 0$

$\sin(\frac{\pi}{2}) = 1$.

Find

$\cos(3\pi)$, $\sin(3\pi)$.

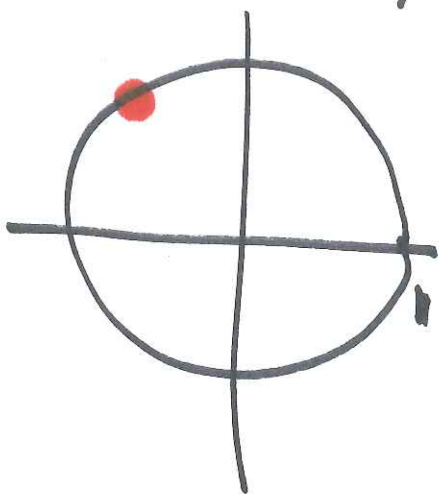


$\cos(3\pi) = -1$

$\sin(3\pi) = 0$

$P(\theta) = (-1, 0)$.

~~Pty~~ Pythagoras.

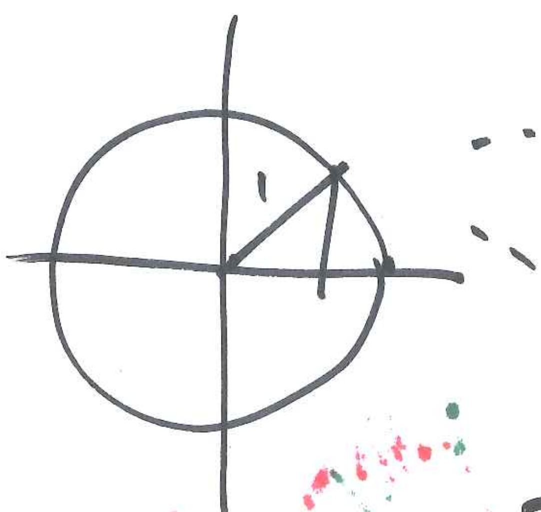


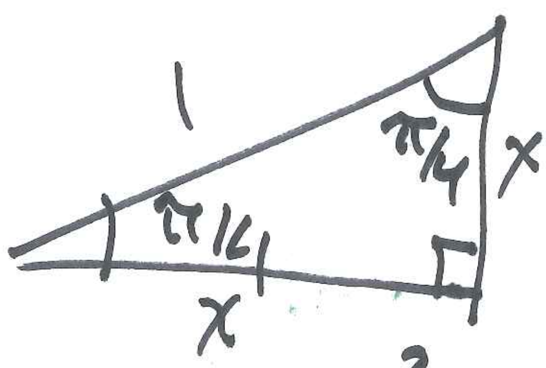
P(θ) is on the unit circle so

$$\sin^2\theta + \cos^2\theta = 1.$$

Special angles.

$\pi/4$ Find $\cos(\frac{\pi}{4})$, $\sin(\frac{\pi}{4})$.





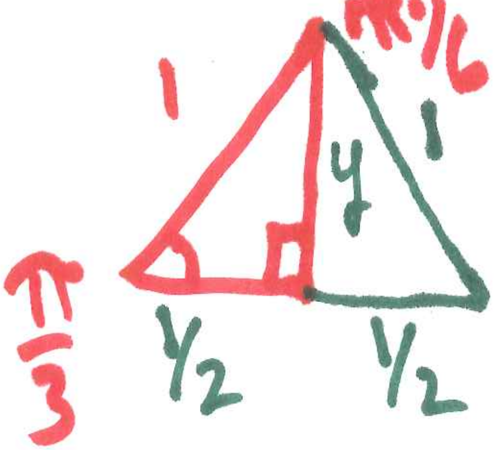
$x^2 + x^2 = 1$, $x^2 = 1/2$

$x = \pm 1/\sqrt{2}$, $x = \frac{1}{\sqrt{2}}$

$$\cos(\pi/4) = \frac{1}{\sqrt{2}} = \sqrt{2}/2$$

$$\sin(\pi/4) = \sqrt{2}/2.$$

Find $\cos(\pi/3)$, $\sin(\pi/3)$



The big triangle is equilateral

Pythagoras tells us

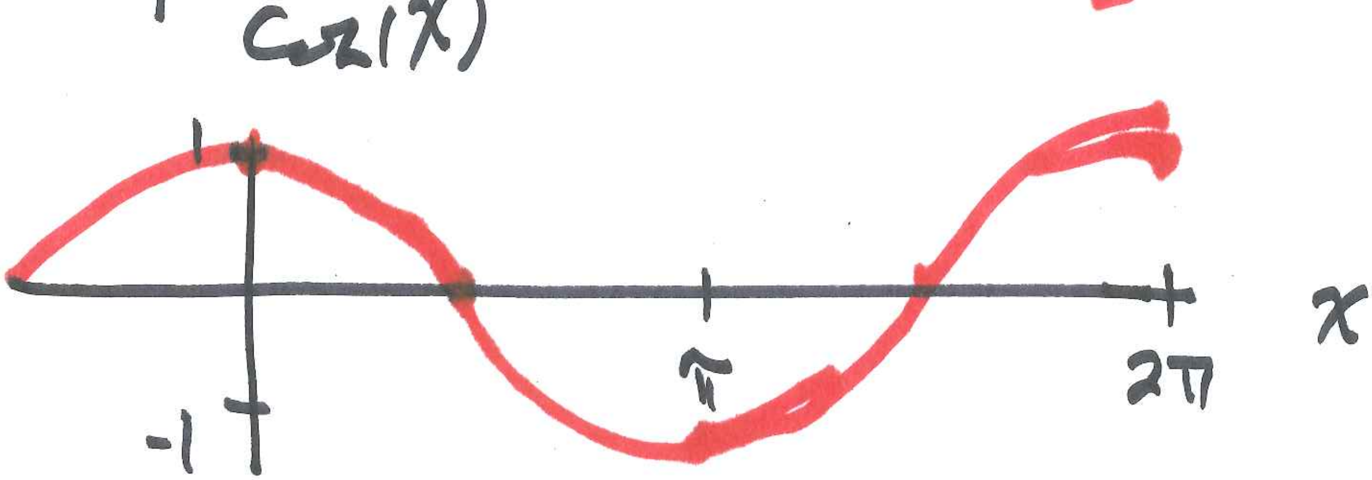
$$1 = \frac{1}{2^2} + y^2, \quad y^2 = 3/4,$$

$$y = \sqrt{3}/2.$$

$$\cos(\pi/3) = \frac{1}{2}, \quad \sin(\pi/3) = \sqrt{3}/2$$

Graph of cos.

θ	0	$\pi/6$	$\pi/4$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{5\pi}{6}$	π
cos	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$1.$



More trig functions.

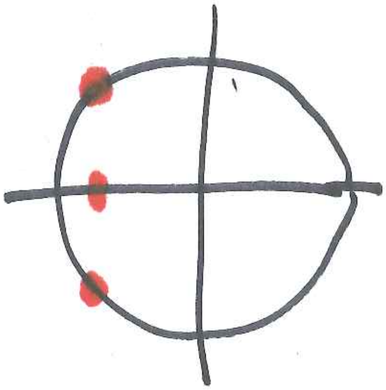
$$\tan(x) = \cancel{\frac{\cos(x)}{\sin(x)}}.\quad \frac{\sin(x)}{\cos(x)}$$

$$\cot(x) = \frac{1}{\tan(x)} = \cancel{\frac{\sin(x)}{\cos(x)}} \quad \frac{\cos(x)}{\sin(x)}$$

$$\sec(x) = \frac{1}{\cos(x)}, \quad \csc(x) = \frac{1}{\sin(x)}.$$

If $\cos(\theta) = -3/5$

and $\frac{\pi}{2} < \theta < \pi$, find $\sin(\theta)$.



$$\cos^2\theta + \sin^2\theta = 1$$

$$\left(-\frac{3}{5}\right)^2 + \sin^2\theta = 1.$$

$$\sin^2\theta = 1 - \frac{9}{25}$$

$$= 16/25.$$

$$\sin(\theta) = +\frac{4}{5} \cancel{\text{or } -4/5}$$

since $\theta \in [\pi/2, \pi]$. so

$\sin(\theta) > 0$

Inverse trig functions

- Need to restrict domain.

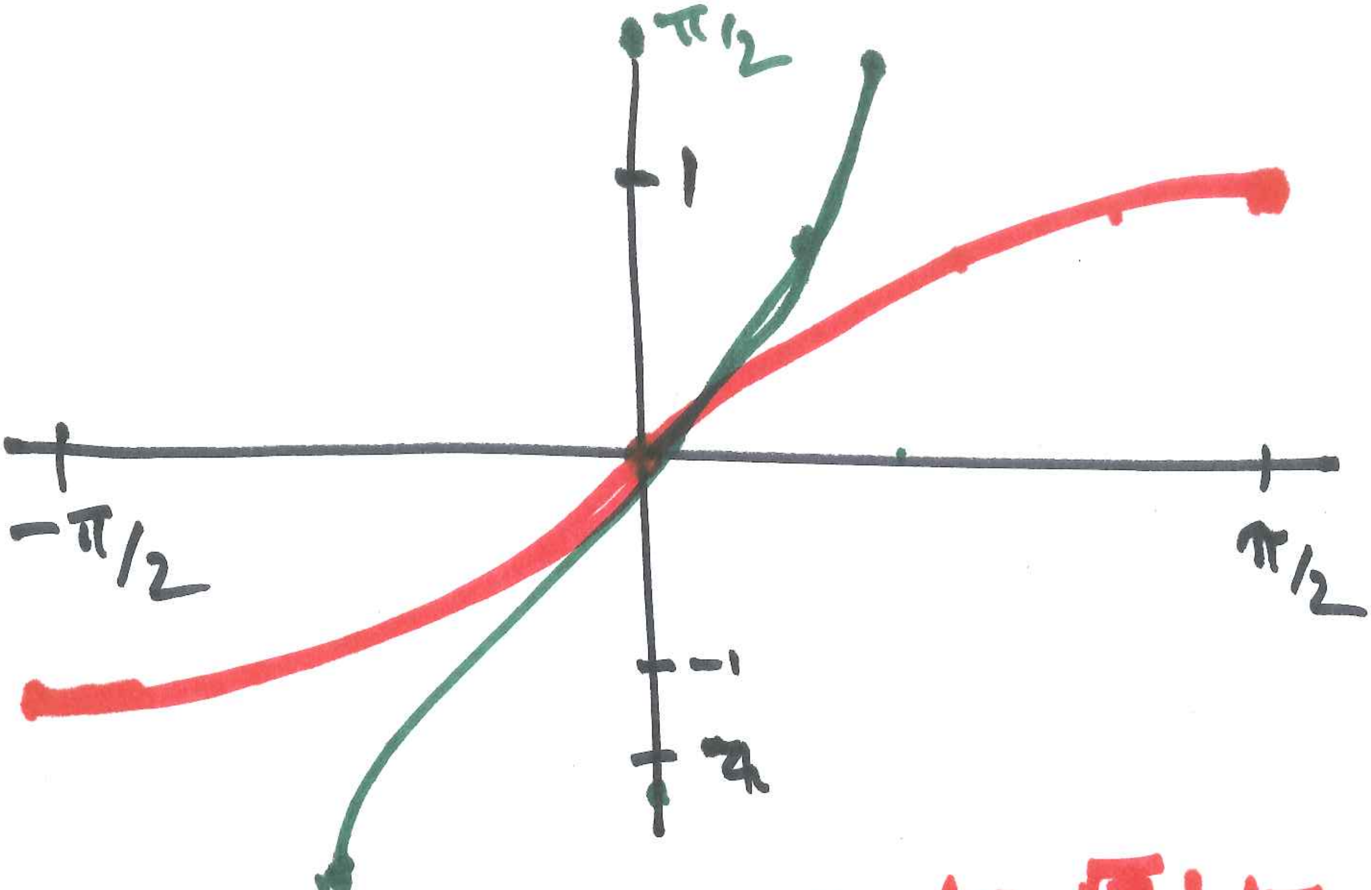
$\cos(x)$ use $[0, \pi]$

$\sin(x)$ use $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

~~Graph~~

We let $\sin^{-1}$ or arcsin and $\cos^{-1}$ or arccos <u>be the</u> inverse functions.

- Graph $\sin^{-1}$.
- Find domain & range.



$y = \sin(x)$. $(0,0)$, $\left(\frac{\pi}{4}, \frac{\sqrt{2}}{2}\right)$ $\left(\frac{\pi}{2}, 1\right)$.

$y = \sin^{-1}(x)$. $(0,0)$. $\left(\frac{\sqrt{2}}{2}, \frac{\pi}{4}\right)$. $\left(1, \frac{\pi}{2}\right)$.

	$[-\pi/2, \pi/2]$	$[-1, 1]$
$\sin(x)$.	Domain	Range.
$\sin^{-1}(x)$	range	domain.