Office hours in Mathsheller today 10-11.

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Exponential & Log.

- Exponential function

$a > 0$, $a \neq 1$.

$n = 1, 2, \ldots$ –

$$a^n = \underbrace{a \cdot a \cdots a}_{n \text{ times}}$$

$$a^{n+m} = a^n \cdot a^m.$$

$a^0 = 1$ (so that $a^0 a^n = a^n$).

$a^{-n} = 1/a^n$ (so that $a^n \cdot a^{-n} = a^0$).

$a^{1/n} = \sqrt[n]{a}$ so that

$$\cancel{(a^{1/n})^n = a^{n}} \; a$$

$$(a^{1/n})^n = a^{1/n} \cdot a^{1/n} \cdots a^{1/n}$$
$$= a^{n/n} = a^1 = \underline{a}.$$

$$(a^n)^m = \underbrace{a^n \cdots a^n}_{m} = a^{n \cdot m}.$$

– Using these rules we can define $a^{n/m}$ for $n, m$ whole numbers $m \neq 0$.

$a^x$ if $x$ is real $x = \sqrt{2}$?

– This is question we will answer this semester.

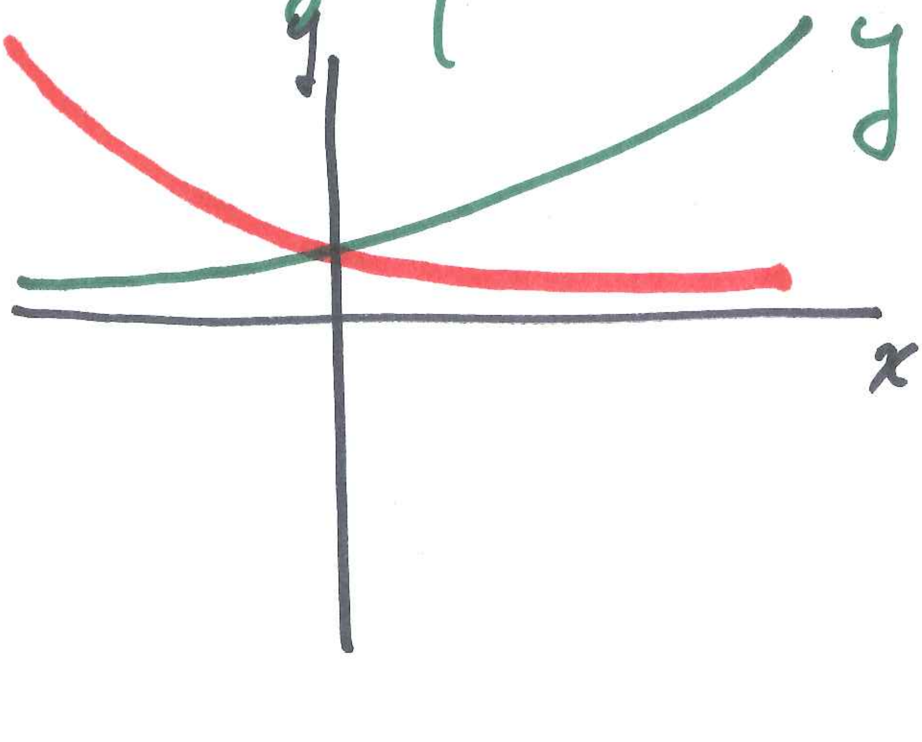
Assume there are functions $a^x$, exponential function with base $a$. Satisfying

$a^x a^y = a^{x+y}$ $\quad a^0 = 1,\ a^{-x} = 1/a^x$

$a^1 = a$ $\quad (a^x)^y = a^{xy}$.

Domain of $a^x$ $\quad (-\infty, \infty)$.

Range of $a^x$ $\quad (0, \infty)$.




$y = a^x,\ a > 1.$

$y = a^x\ 0 < a < 1.$

Logarithm with base $a$. $a>0$, $a\neq 1$.

$\log_a x$ is the inverse function to $a^x$.

$\log_{10} 100 = 2$ since $10^2 = 100$.

In general

$y = \log_a(x)$ if $x = a^y$.

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Find

$\log_2 \sqrt{2}$, $\log_a a$, $\log_a 1$

$= 1/2$ $\quad = 1$

$\log_a 1 = 0$ since $a^0 = 1$.

Properties of $\log_a$.

$\log_a x + \log_a y = \log_a (xy)$.

$\log_a 1/x = -\log_a x$

$\log_a x^r = r \log_a x$.

(on $x > 0$, $y > 0$ and $r$ in $(-\infty, \infty)$).

Why?

$\log_a x + \log_a y$

Know $a^{\log_a x} = x$

$a^{\log_a y} = y$

$xy = a^{\log_a x} \cdot a^{\log_a y}$

$= a^{\log_a x + \log_a y}$

So $\log_a(xy) = \log_a x + \log_a y.$

So many logarithms! 7

- Which one is best?

- We use the base $e = 2.7818 \ldots$

- Why? ~~Formato~~ Calculus works well with this one.

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Notation $y = \ln(x)$.

for log to base $e$ or the natural logarithm.

we write

$$e^x = \exp(x).$$

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why do you care?

- Solve equations
- Exp. Growth & Decay.

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Solve

$$10^{x^2} = 7.$$

$$\log_{10} 10^{x^2} = \log_{10} 7.$$

$$x^2 \log_{10} 10 = \log_{10} 7.$$

$$x^2 \cdot 1 = \log_{10} 7.$$

$$x = \pm\sqrt{\log_{10} 7}.$$

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Solve

$$2^{2x+1} = 3$$

Apply $\ln$

$$\ln(2^{2x+1}) = \ln(3).$$

$$\frac{(2x+1)\ln(2)}{\ln(2)} = \frac{\ln(3)}{\ln(2)}.$$

$$2x \cancel{+1} = \frac{\ln(3)}{\ln(2)} - 1.$$

$$x = \frac{1}{2}\left(\frac{\ln(3)}{\ln(2)} - 1\right).$$

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<u>Example</u> Write $4^x$ in the form $e^{rx}$

<u>Solution</u>

$$y = 4^x = e^{\ln(4^x)}.$$

$$= e^{x\ln(4)}.$$

If $\ln(xy^2) = 2$

and $\ln(x/y) = 0$

Find $x$ & $y$.

<u>Solution</u> Props of log.

(1) $\ln(xy^2) = \ln(x) + 2\ln(y) = 2$

(2) $\ln(x/y) = \ln x - \ln y) = 0.$

System of equations/or

$\ln(x)$ & $\ln(y)$

$+ 3\ln(y) = 2.$

By subtracting (2) from (1)

so $\ln(y) = 2/3$

$$\underline{y = e^{2/3}}$$

$\ln(x) = \ln(y)$ by eq. (2)

$\underline{x = e^{2/3}}$ also