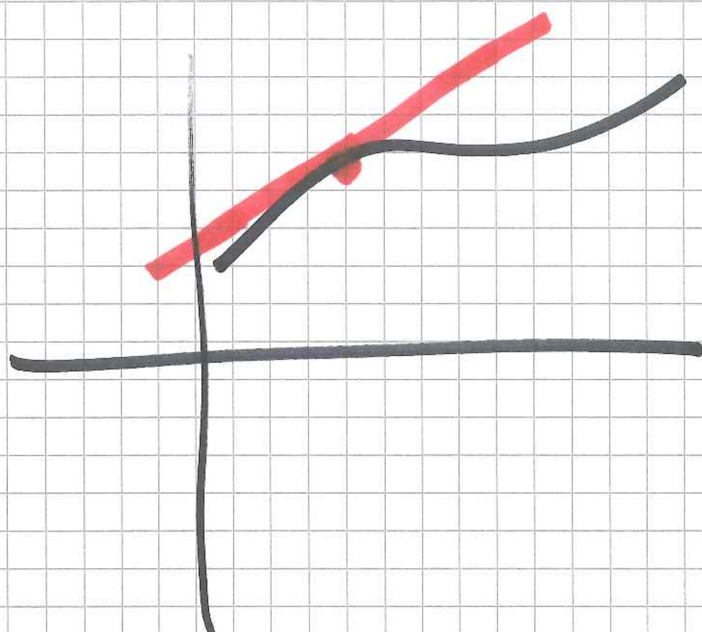
① 9/2/15

$y = f(x)$, find tangent line.



Example $f(x) = e^x$

Find tangent line at $x = 0$

— Need point & slope

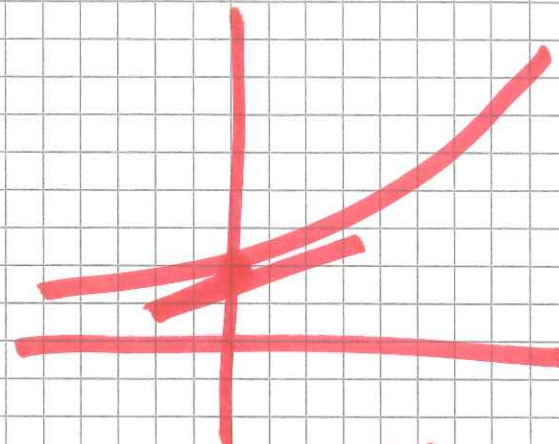
Line thru (x_0, y_0), slope m

$$\boxed{y - y_0 = m(x - x_0)}$$

Point is (0,1).

Slope?

$m = \frac{1-1}{0-0} = \frac{0}{0}$?!



Try again using (0,1)
and a nearby point $(0.1, f(0.1))$

$m = \frac{f(0.1) - f(0)}{0.1 - 0}$ is the
slope thru $(0, f(0))$ to
$(0.1, f(0.1))$.

x	m
0.1	1.05
0.01	1.005
-0.002	0.999
$1/\pi^3$	1.0163

3 Guess – As x gets closer to zero slope gets closer to 1.

Tangent line has slope 1 & passes thru (0,1). Equation is

~~$y - 1 = m$~~

$$y - 1 = 1 \cdot (x - 0)$$

$$y = x + 1.$$

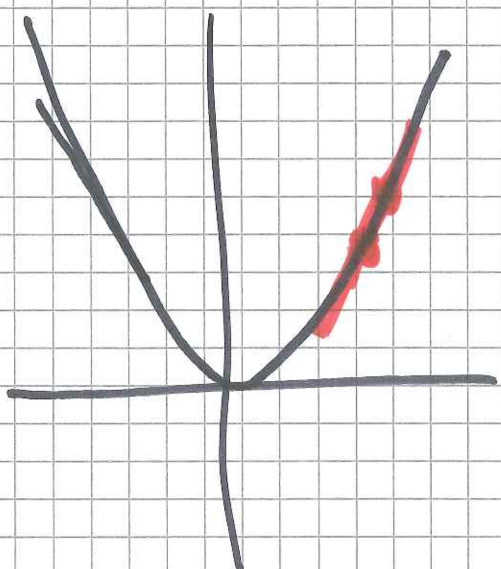
Summary of procedure to find tangent line to $y = f(x)$ at x_0

1. line thru $(x_0, f(x_0))$
2. Find slope by computing slope of the secant line thru $(x_0, f(x_0))$ & $(x_0+h, f(x_0+h))$

$$m = \frac{f(x_0+h) - f(x_0)}{h}$$

Let h approach zero to guess what happens.

Find Slope of secant line to $y = x^2 = f(x)$ for $x = 2$ to 2.1



$$\frac{(2.1)^2 - 2^2}{2.1 - 2} = \frac{4.41 - 4}{0.1} = \underline{\underline{4.1}}$$

Limits

If the values of $f(x)$ are close to L when $x \neq a$ is close to a, we say that $\lim_{x \to a} f(x) = L$.

We say the limit as x approaches a of $f(x)$ is L.

We saw

$$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

Example find slope to $f(x) = x^2$ at $x = 2$.

Consider $\dfrac{f(x) - f(2)}{x - 2}$

$$= \frac{x^2 - 4}{x - 2} = \frac{\cancel{(x-2)}(x+2)}{\cancel{x-2}}$$

$$\lim_{x\to 2}\frac{f(x)-f(2)}{x-2}=\lim_{x\to 2}(x+2)$$

$$=\underline{\underline{4}}$$

~~Find~~

Velocity

A particle moves along a line. The position is given by a function $p(t)$.

— Average velocity on an interval $[a,t]$ is $\dfrac{p(t)-p(a)}{t-a}$

Instantaneous velocity at time a is

$$\lim_{t \to a} \frac{p(t) - p(a)}{t-a} = v(a).$$

Example $f(x) = |x-1|$

Tangent line at $x = 1$?

Try $\dfrac{f(1+h) - f(1)}{1+h-1}$

$= \dfrac{|h| - 0}{h} = \dfrac{|h|}{h}.$

$$\frac{|h|}{h} = \begin{cases} 1, & h>0 \\ -1, & h<0. \end{cases}$$

undef. if $h=0$

As h approaches
the slope does not
approach a number.

$\lim_{h \to 0} \frac{|h|}{h}$ does not exist.

$|h|/h$.