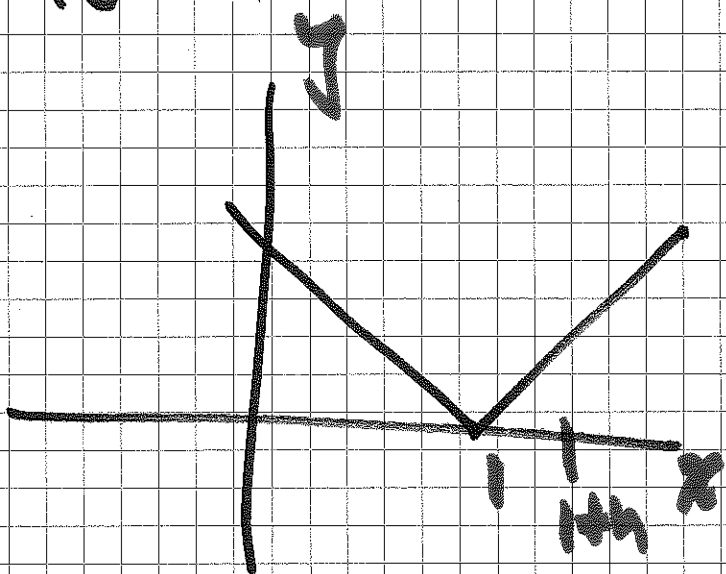
9/5/15

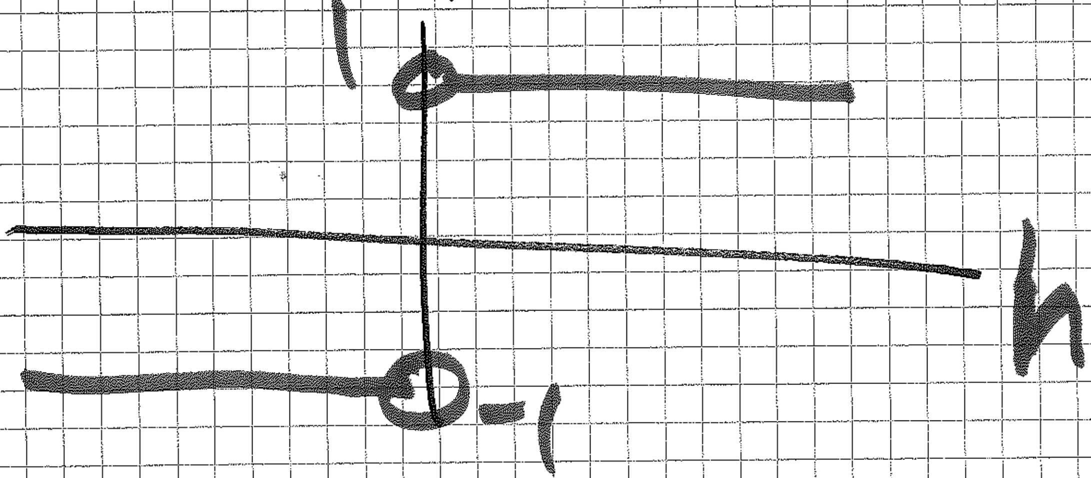
Slope of $|x-1|$ at $x = 1$.



$$\frac{f(1+h)-f(1)}{h} = \frac{|h|}{h} = \begin{cases} 1, & h>0 \\ -1, & h<0 \end{cases}$$

$\lim_{h\to 0} \frac{|h|}{h}$ does not exist.

Graphing $\frac{|h|}{h}$.



One-sided Limits

Let f be defined for x > a and near a.

If f(x) is close to L when x is close to a and near a, then

$$\lim_{x \to a^+} f(x) = L.$$

Also define $\lim_{x \to a^-} f(x)$.

. . . .

— o — we have

<u>Fact</u>. ~~The~~ $\lim_{x \to a} f(x) = L$.

is equivalent to

$\lim_{x \to a^+} f(x)$ and $\lim_{x \to a^-} f(x)$ exist

and are equal to L.

— o —

<u>Example</u> Does

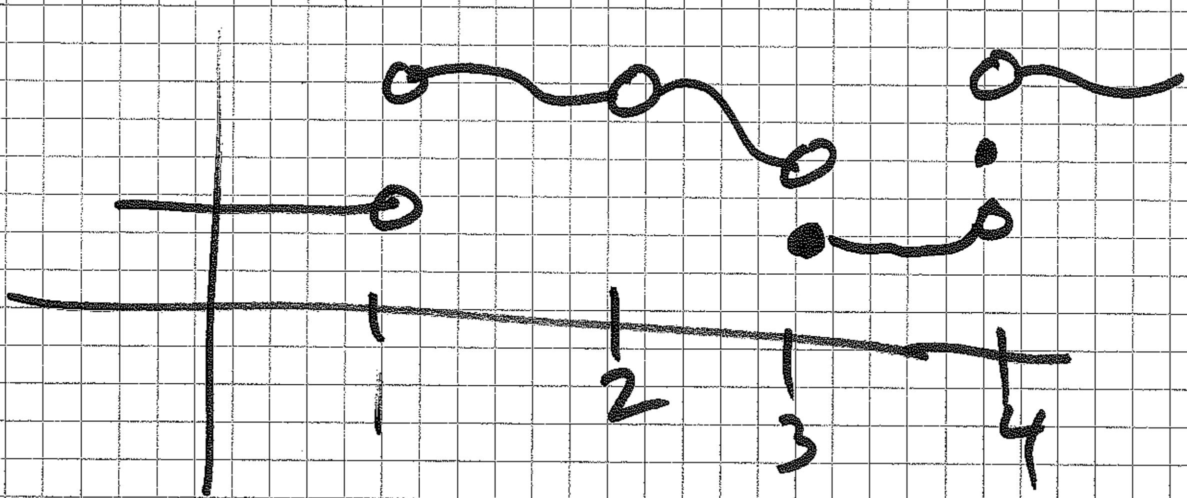
$\lim_{h \to 0} \frac{|h|}{h}$ exist?

No: $\lim_{h \to 0^+} \frac{|h|}{h} = 1$ and

$\lim_{h \to 0^-} \frac{|h|}{h} = -1$.

Since the one-sided limits are different, the two-sided limit does not exist.

Example. Consider



If $f(x) = \dfrac{x^2-4}{x+2}$.

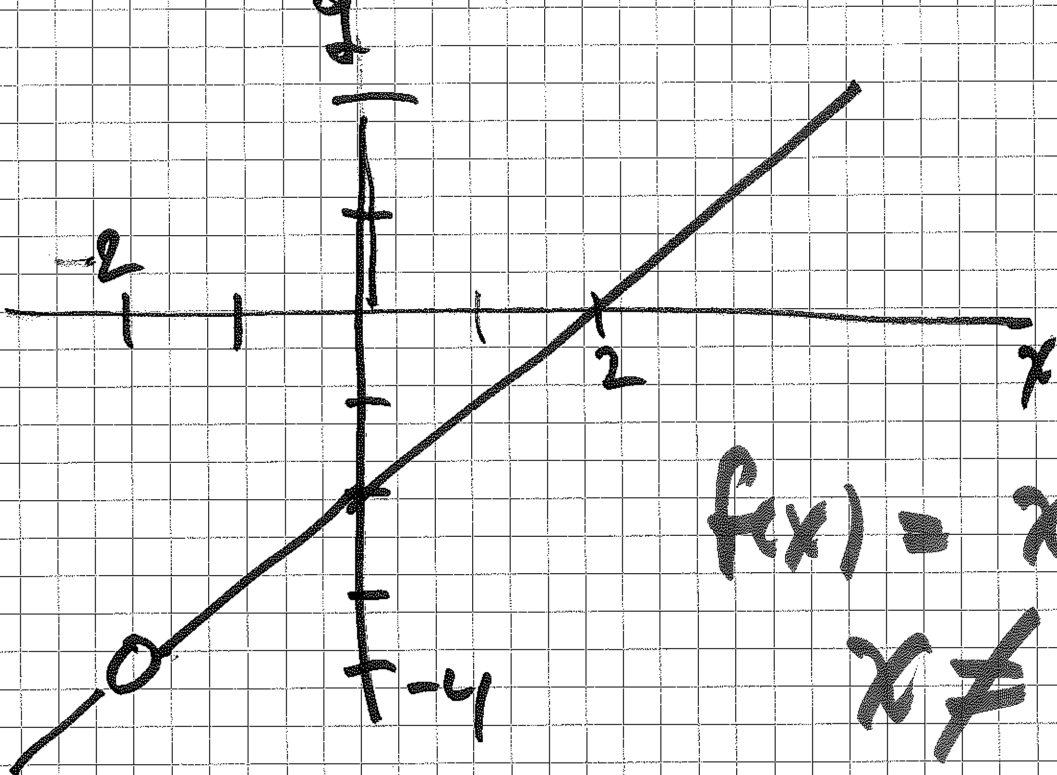
- Graph f
- $\lim_{x\to -2} f(x)$.

Simplify $\dfrac{x^2-4}{x+2} = \dfrac{(x-2)\cancel{(x+2)}}{\cancel{x+2}}$

$= x-2$, if $x \neq -2$.

$\dfrac{x^2-4}{x+2}$ is undefined if

$x = -2$.



$f(x) = x-2$ if
$x \neq -2$.

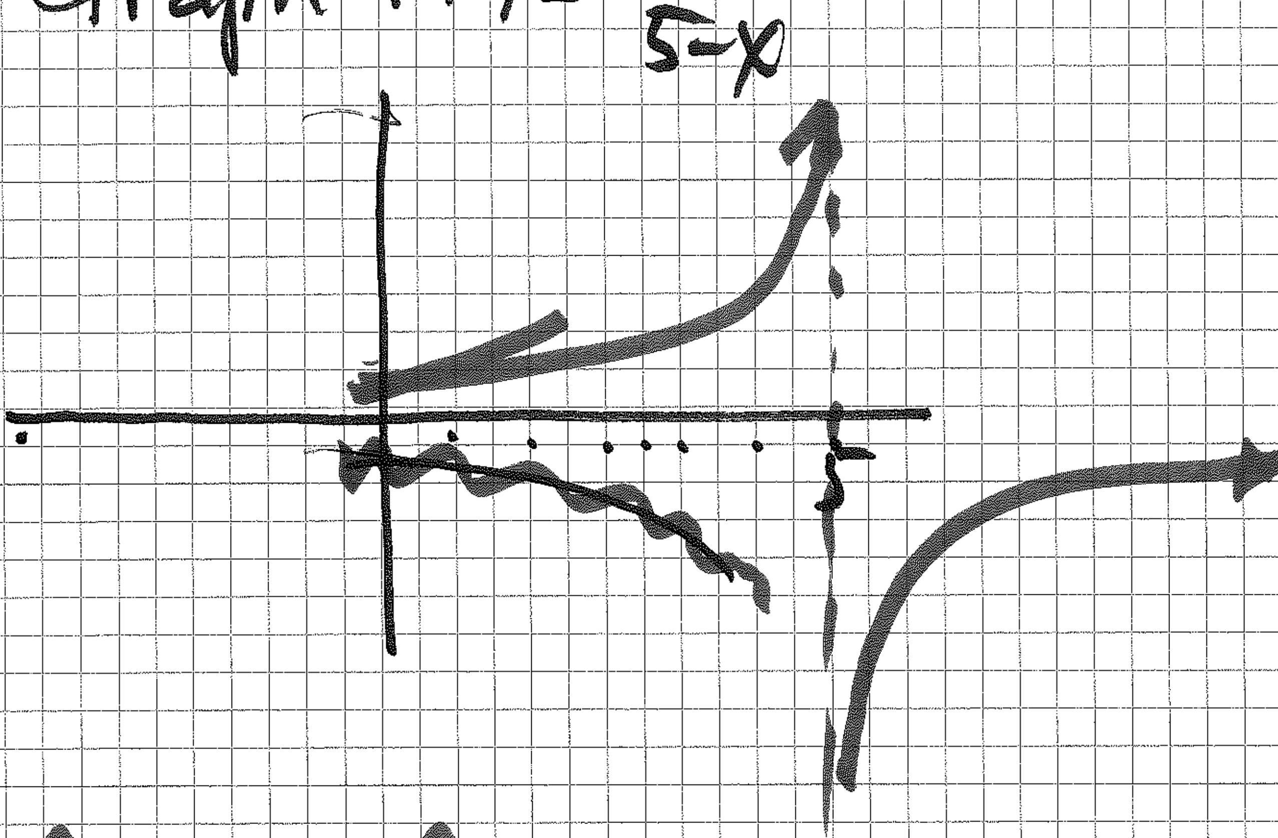
$f(-2)$ is undefined
But $\lim_{x \to -2} f(x) = \underline{-4}$

Limits that are infinite

<u>Example</u>

$$f(x) = \frac{1}{5-x}$$

Graph $f(x) = \frac{1}{5-x}$



f gets large and positive as x approaches 5 with $x<5$!

f is large negative as x approaches 5 with $x>5$.

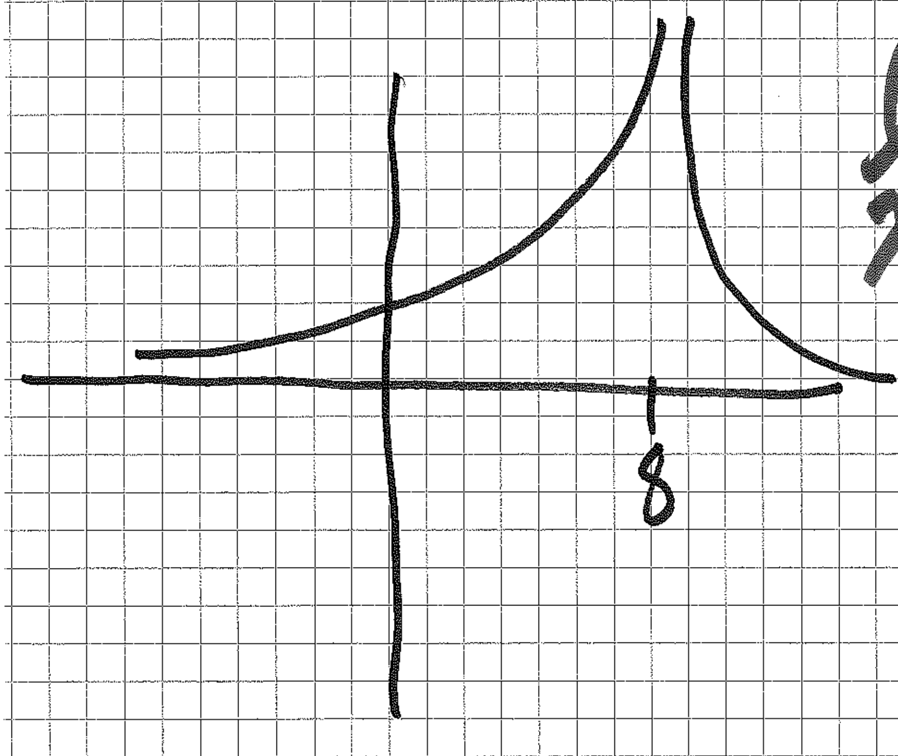
we say $\lim_{x \to 5^-} f(x) = +\infty$

and $\lim_{x \to 5^+} f(x) = -\infty$.

$\lim_{x \to 5} f(x)$ does not exist.

Example $f(x) = \dfrac{1}{(x-8)^2}$

$\lim_{x \to 8} f(x) = +\infty$



Computing limits.

Two basic limits

$$\lim_{x \to a} x = a \qquad \lim_{x \to a} c = c$$

Rules. If $\lim_{x \to a} f(x) = L$

and $\lim_{x \to a} g(x) = M$.

then

$$\lim_{x \to a} (f(x) + g(x)) = L + M.$$

$$\lim_{x \to a} (f(x) g(x)) = L \cdot M.$$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{L}{M} \text{ if } M \neq 0$$

$$\lim_{x \to a} \sqrt[n]{f(x)} = L^{1/n},$$

if $L > 0$ when ~~$a \geq 0$~~ n is even

$$\lim_{x \to a} f(x)^{m/n} = L^{m/n}$$

where $L > 0$
when n is even

Examples

$$\lim_{x \to 2} (3x^2 + 2)$$

10.5

Using rule for limits of a product,

$$\lim_{x \to 2} 3x^2$$

$$= \lim_{x \to 2} 3 \lim_{x \to 2} x \cdot \lim_{x \to 2} x$$

$$= 3 \cdot 2 \cdot 2 = \underline{12}$$

By rule for a sum of limits,

$$\lim_{x \to 2} (3x^2 + x)$$

$$= \lim_{x \to 2} 3x^2 + \lim_{x \to 2} x$$

$$= 12 + 2 = \underline{14}$$

$$\lim_{x \to 2} \frac{x^2-4}{x+2} =$$

$$\lim_{x \to 2} x^2-4 = 0$$

$$\lim_{x \to 2} x+2 = 4.$$

$$\lim_{x \to 2} \frac{x^2-4}{x+2} = \frac{0}{4}$$ by

rule for limit

of a quotient.

$$\lim_{x \to -2} \frac{x^2-4}{x+2}$$

$$\lim_{x \to -2} x^2-4 = 0$$

$$\lim_{x \to -2} x+2 = 0$$

Can't use rule for limit of quotients.

$$\frac{x^2-4}{x+2} = \frac{(x-2)\cancel{(x+2)}}{\cancel{x+2}}$$

13

$$\lim_{x \to -2} \frac{x^2-4}{x+2}$$

$$= \lim_{x \to -2} (x-2).$$

$$= \underline{\underline{-4}}$$