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$$\lim_{x \to 1} \frac{x^2-1}{x^3-1}$$

$$x^2-1 = (x-1)(x+1).$$

$$x^3-1 = (x-1)(x^2+x+1).$$

$$\frac{x^2-1}{x^3-1} = \frac{\cancel{(x-1)}(x+1)}{\cancel{(x-1)}(x^2+x+1)}$$

$$= \frac{x+1}{x^2+x+1}, \quad x \neq 1.$$

Can't use limit law for quotients to evaluate $\lim\limits_{x \to 1} \frac{x^2-1}{x^3-1}$ since $\lim\limits_{x \to 1} x^3-1 = 0$

$\lim_{x \to 1} \frac{x+1}{x^2+x+1}$ allows the quotient rule since

$$\lim_{x \to 1} (x^2+x+1) = 3.$$

$$\lim_{x \to 1} (x+1) = 2$$

So $\lim_{x \to 1} \frac{x+1}{x^2+x+1} = \frac{2}{3}.$

Continuous functions.

Recall if p is a polynomial, and c a number

$$\lim_{x \to c} p(x) = p(c), \text{ always}$$

Rational functions.

$$R(x) = \frac{P(x)}{Q(x)} \quad \text{where } P, Q$$

are polynomials - then

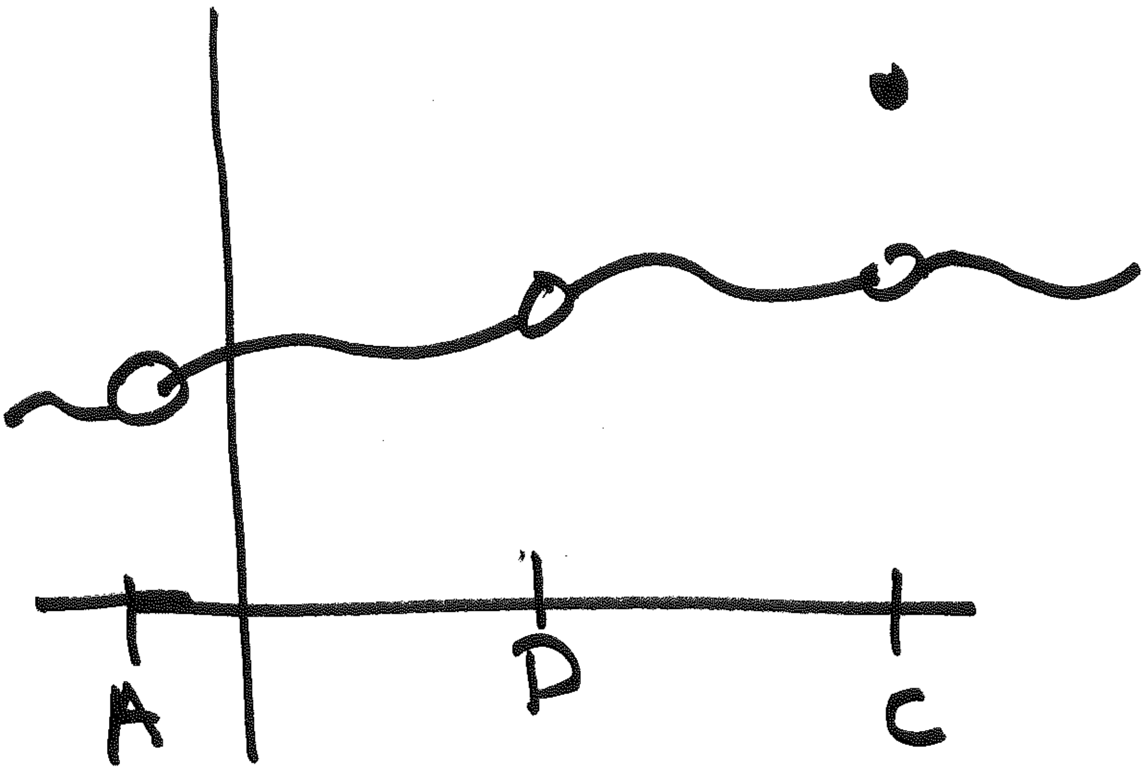
$$\lim_{x \to c} R(x) = R(c) \text{ if } c \text{ is}$$

in the domain of $R(x)$.

<u>Def.</u> A function f defined on an open interval containing a is containing a is continuous at a if

$$\lim_{x \to a} f(x) = f(a).$$

- $f(a)$ exists.
- $\lim_{x \to a} f(x)$ exist
- They are equal.



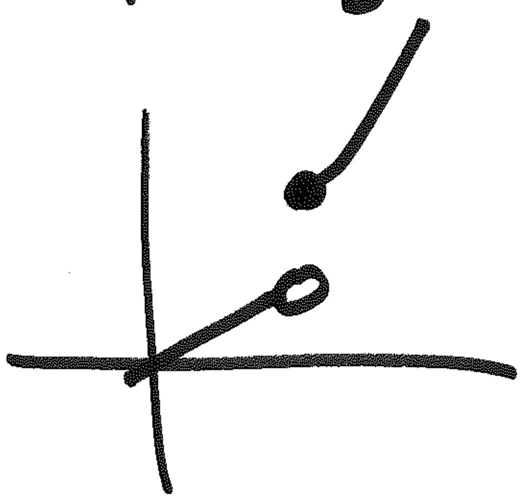
Def. If f is defined on an interval w/ left end point a. and $\lim_{x \to a^+} f(x) = f(a)$, then we say f is right-continuous at a

Left continuous a. – Imagine the answer.

Example. Let

$$f(x) = \begin{cases} x^2 & , x \geq 1 \\ kx & , x < 1. \end{cases}$$

Find k so that f is continuous everywhere.



$$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} x^2$$

$$= 1.$$

$$\lim_{x \to 1^-} f(x) = \lim_{x \to 1} kx$$

$$= k.$$

$\lim_{x \to 1} f(x)$ exists if

$k = 1$

and then

$$\lim_{x \to 1} f(x) = 1 = f(1).$$

So f is continuous at 1.

~~We know~~

List of continuous functions.

- Polynomials.
- Rational functions (on their domains).
- Powers x^a
- Exponential a^x
- logarithms
- trig, inverse trig.

Substitution rule.

If f is continuous at a, then we can ~~not~~ evaluate $\lim_{x \to a} f(x)$ by substitution

$$\lim_{x \to a} f(x) = f(a).$$

Example

Example. $\lim\limits_{x \to 0} \tan(x)\csc(x)$.

$\tan(0)\csc(0)$ is undefined

$$\tan(x)\csc(x) = \frac{\sin(x)}{\cos(x)}\frac{1}{\sin(x)}$$

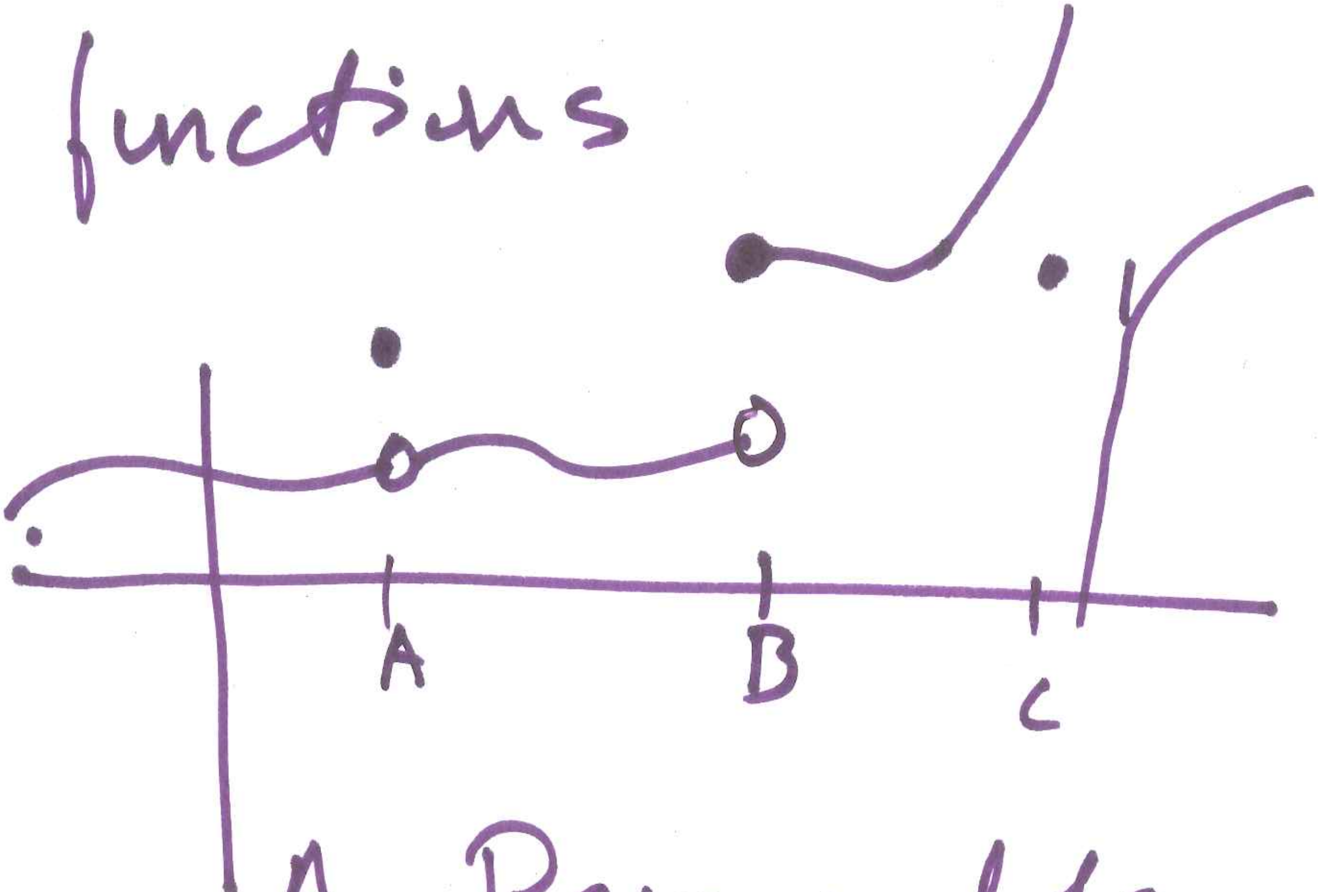
$$= \frac{1}{\cos(x)}$$

$\frac{1}{\cos(x)}$ is continuous at 0

So $\lim\limits_{x \to 0} \frac{1}{\cos(x)} = \frac{1}{1} = 1.$

by substitution.

Discontinuous functions



A - Removeable

B - Jump discontinuity

C - Infinite discontinuity.