9/14/15

$$\lim_{x\to 0} x^2 \sin\left(\frac{1}{x^2}\right).$$

- Use product rule for limits? — <u>No</u>

$$\lim_{x\to 0} x^2 = 0$$

$$\lim_{x\to 0} \sin\left(\frac{1}{x^2}\right) = \cancel{0}\ \text{DNE}$$

Squeeze theorem -

Suppose f, g, h are functions defined for x near a, but not necessarily for $x = a$.

Suppose $f(x) \leq g(x) \leq h(x)$ for x near a. Suppose

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L.$$

Then $\lim_{x \to a} g(x) = L$.

<u>Example</u> Show

$$\lim_{x \to 0} x^2 \sin(1/x^2) = 0$$

We have

$$-x^2 \leq x^2 \sin(1/x^2) \leq +x^2$$

for all $x \neq 0$.

Also $\lim\limits_{x\to 0} x^2 = \lim\limits_{x\to 0} -x^2 = 0$

So then by the squeeze theorem

$$\lim_{x\to 0} x^2 \sin(1/x^2) = 0.$$

Turning point.

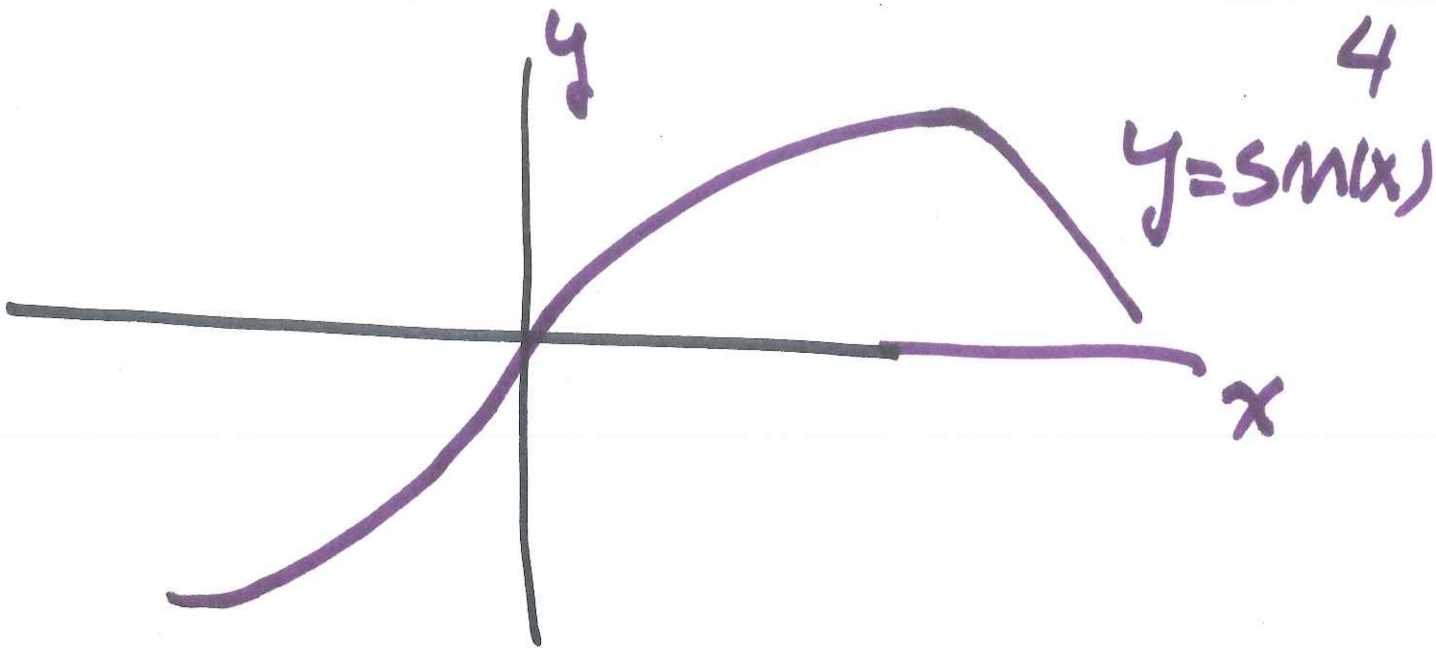
$$-x^2+4x-3 \le f(x) \le x^2 \overset{-}{\cancel{+}} 4x+5.$$

$$-(x^2-4x+4-4)-3$$

$$-(x-2)^2+4-3 = -(x-2)^2+1.$$

$$\cancel{x^2 2 -} \quad x^2-4x+5 = (x^2-4x+4-4)+5$$

$$= (x-2)^2-4+5 = (x-2)^2+1.$$

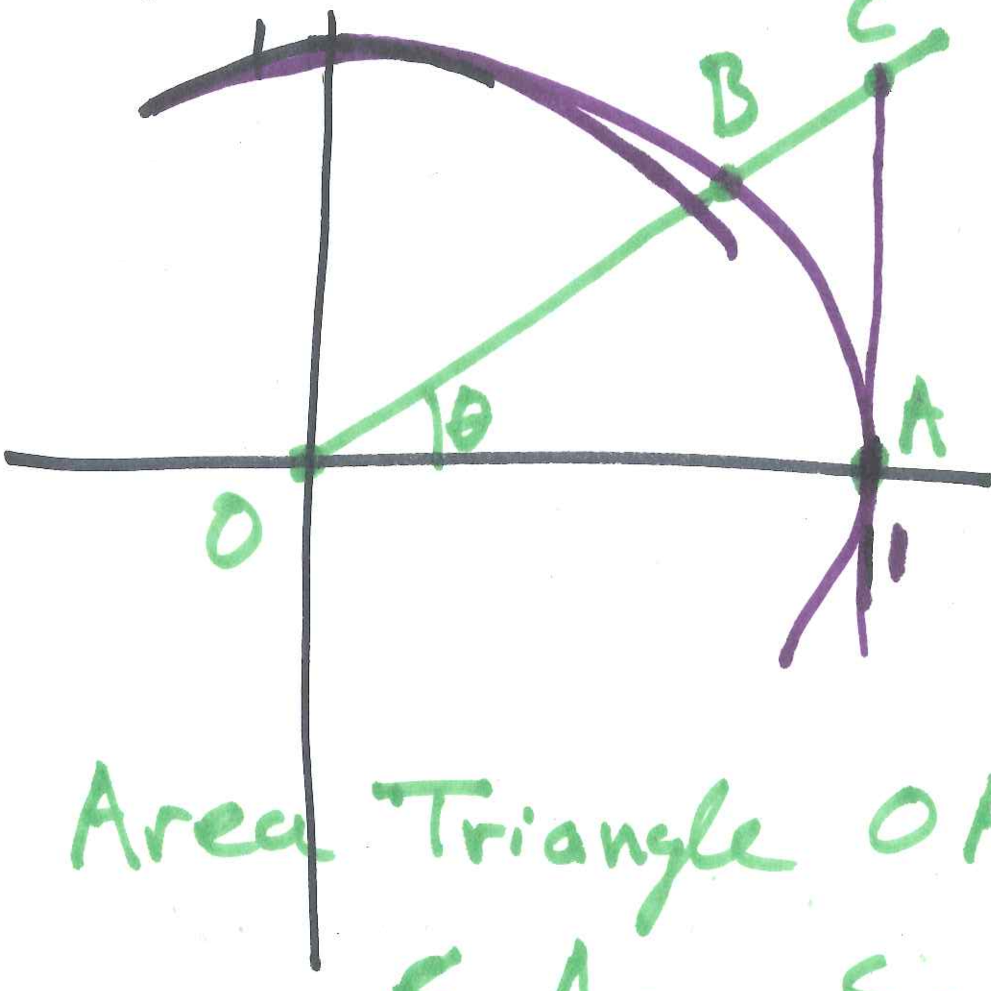


Find slope of tangent line at 0

$$\lim_{x \to 0} \frac{\sin(x) - \sin(0)}{x - 0} = \lim_{x \to 0} \frac{\sin(x)}{x}.$$

— Guess by trying numbers

Better approach. Use squeeze theorem.



Area Triangle OAB.

$\leq$ Area Sector OAB

$\leq$ Area triangle OAC

Sector has area $\frac{\theta}{2\pi} \cdot \pi = \frac{\theta}{2}$.

Area $OAB = \frac{1}{2}\sin(\theta)$.

$A = (1,0)$. $B = (\cos\theta, \sin(\theta))$.

$O = (0,0)$.

Area $OAC = \frac{1}{2}\tan(\theta)$.

$C = (1, \tan(\theta))$

$$\frac{1}{2}\sin(\theta) \leq \frac{1}{2}\theta \leq \frac{1}{2}\tan(\theta).$$

Thus $\frac{\sin(\theta)}{\theta} \leq 1$, $\theta > 0$

and $\cos(\theta) \leq \frac{\sin(\theta)}{\theta}$, $\theta > 0$.

Altogether

$$\cos(\theta) \leq \frac{\sin(\theta)}{\theta} \leq 1, \ \theta \neq 0.$$

$0 < |\theta| < \pi/2$.

since $\frac{\sin\theta}{\theta}$ is even.

7.

$$\cos\theta \le \frac{\sin\theta}{\theta} \le 1.$$

$$\lim_{\theta \to 0} \cos(\theta) = \lim_{\theta \to 0} 1 = 1.$$

$\therefore$ By squeeze theorem

$$\boxed{\lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} = 1.}$$

$$\lim_{x \to 0} \frac{1-\cos(x)}{x}$$

Solution

$$\frac{1-\cos x}{x} \frac{(1+\cos(x))}{(1+\cos(x))}.$$

$$= \frac{1-\cos^2 x}{x} \frac{1}{1+\cos(x)}$$

$$= \frac{\sin^2(x)}{x} \cdot \frac{1}{1+\cos(x)}.$$

$$= \frac{\sin(x)}{x} \cdot \frac{\sin(x)}{1+\cos(x)}$$

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1.$$

$$\lim_{x \to 0} \frac{\sin(x)}{1+\cos(x)} = 0.$$

$$\lim_{x \to 0} \frac{\sin(x)}{x} \cdot \frac{\sin(x)}{1+\cos(x)} = 0$$

by product rule for limits.

$$\lim_{x \to 0} \frac{\sin(-2x)}{x}$$

$$= \lim_{x \to 0} \frac{\sin(-2x)}{-2x}(-2).$$

$$= -2 \lim_{x \to 0} \left(\frac{\sin(-2x)}{(-2x)}\right)$$

$$= -2 \lim_{(-2x) \to 0} \frac{\sin(-2x)}{-2x}$$

$$= -2 \lim_{u \to 0} \frac{\sin(u)}{u}$$

$$= -2 \cdot 1 = \underline{-2}$$