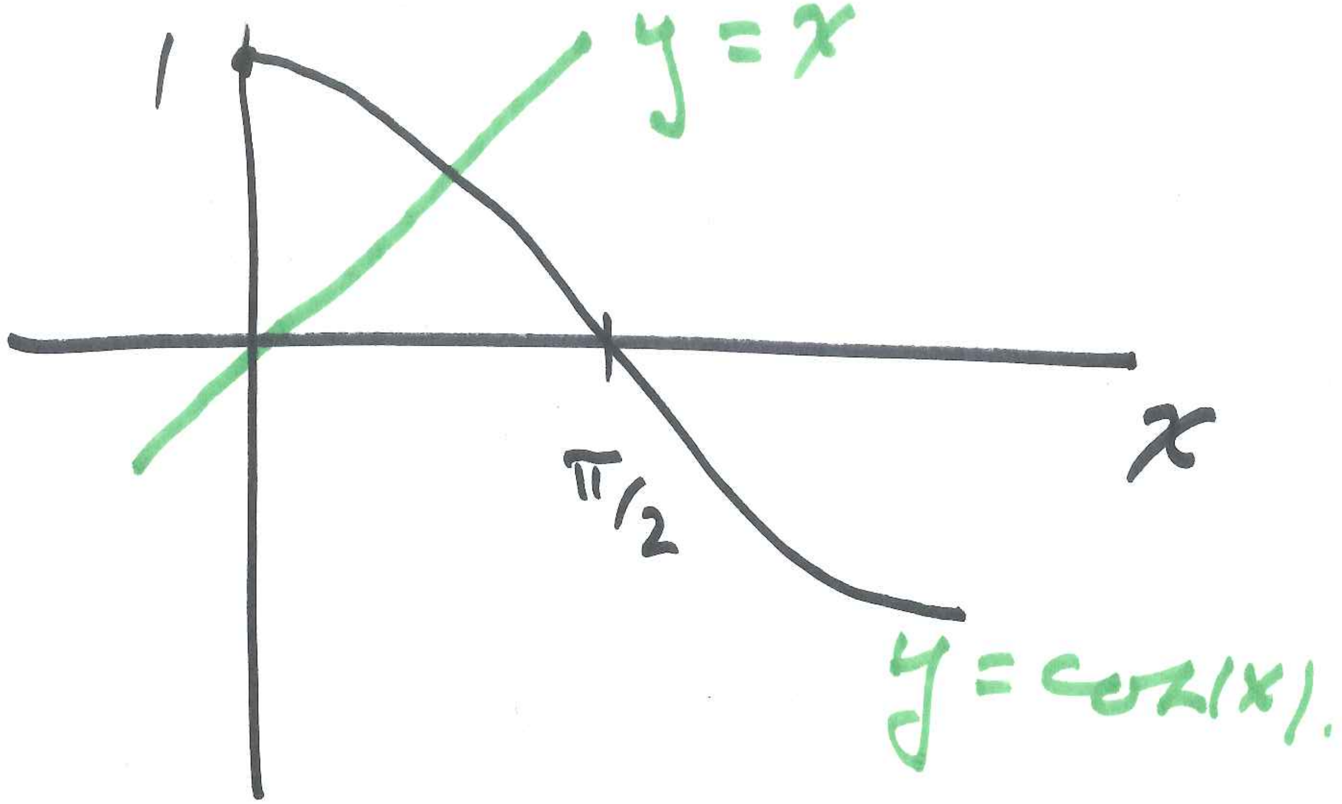
1 9/16/15.

- Alternate exams — I will ~~as~~ reply tonight.
- Response Ware. Send email to let me know you missed the first question.

Intermediate value theorem.

- Does the equation $\cos(x) = x$ have a solution?



Rewrite equation as

$x - \cos(x) = 0$.

$x = 0 \qquad 0 - \cos(0) = -1 < 0$

$x = \frac{\pi}{2} \qquad \frac{\pi}{2} - \cos(\frac{\pi}{2}) = \pi/2 > 0$.

– Between 0 & $\pi/2$ we cross thru 0.

– Provided $x - \cos(x)$ is continuous.

IVT = Intermediate Value Theorem.

If f is continuous on $[a,b]$ and L lies between $f(a)$ & $f(b)$. then the equation:

$$f(x) = L$$

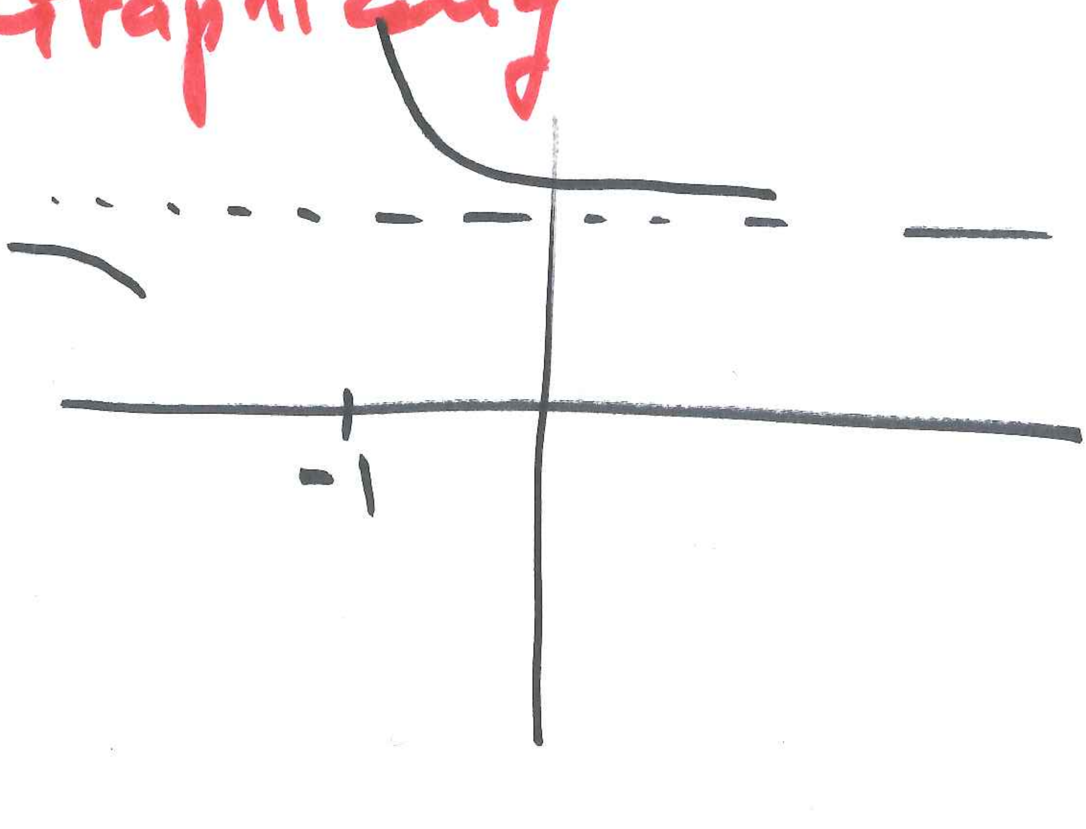
has a solution in (a,b).

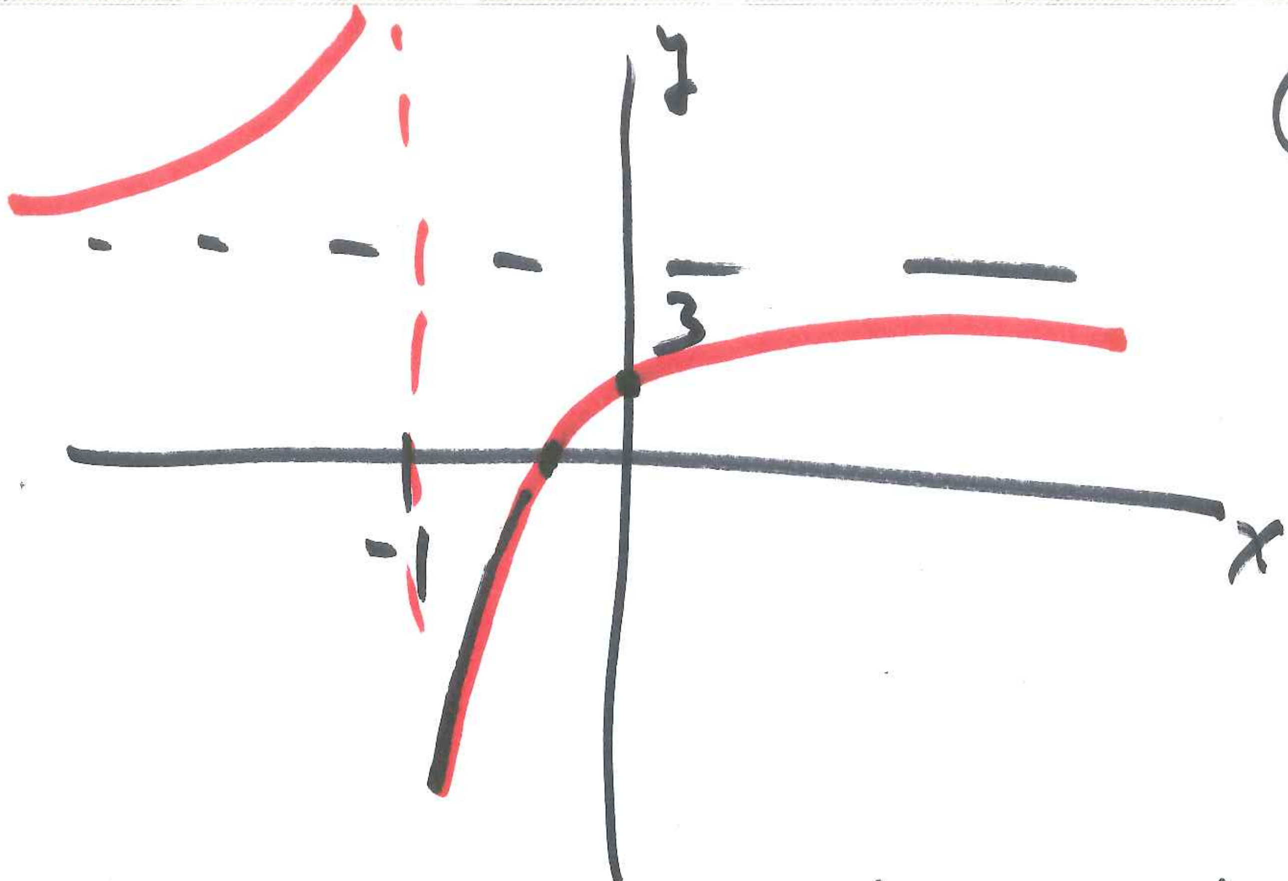
~~Exam~~

Example. Can you use the intermediate value theorem to solve

$$\frac{3x+1}{1+x} = 3 ?$$

Graphically





$y = \frac{3x+1}{1+x}$

x-intercept is $x = -1/3$

y-intercept is $y = 1$.

- No interval $[a, b]$ on which $\frac{3x+1}{1+x} = f(x)$ is continuous,

& 3 lies between

$f(a)$ & $f(b)$

$f(x) = \frac{1}{x}$. $[1,4] = [a,b]$

$f(1) = 1$, $f(4) = 1/4$.

2 is not between 1 & 1/4.

- B is not right.

A is not right since f is not continuous [-1,1].

$f(1/2) = 2$, $f(3) = 1/3$, but

-1 is not between 1/3 & 2.

- Can't use IVT to find a solution to $f(x) = -1$.

On the interval $[\frac{1}{2}, 2]^{2}$.

- Even though there is a solution to $f(x) = -1$, with $x < 0$.

Do square roots exist?

Can you solve $x^2 = 4$?

Can you solve $x^2 = 2$?

$x = 2$ (or -2) solves $x^2 = 4$.

$f(x) = x^2$ is continuous everywhere.

Try $[a,b] = [1,2]$.

then $f(1) = 1.$, $f(2) = 4$

and 2 is between $f(1)$ + $f(2)$. So IVT tells there is a solution to $x^2 = 2$ in the interval $(1, 2)$.

Can you do better?

- Is there a solution in $[1.4, 1.5]$?

Yes $f(1.4) = 1.96$

$f(1.5) = 2.25$

So $\sqrt{2}$ is in $(1.4, 1.5)$.

Bisection method.

Start: Continuous ~~on [~~ function f on $[a,b]$ with ~~L~~ between $f(a)$ & $f(b)$.

Let $m = \frac{a+b}{2}$. Consider $f(a)$, $f(m)$, $f(b)$. L is between $f(a)$ & $f(b)$, therefore L is between either $f(a)$ & $f(m)$ or $f(b)$ & $f(m)$.

So
$$f(x) = L$$
has a solution either ~~on~~ $[a, m]$ or $[m, b]$.

- Repeat until tired.

__Example__. Approximate solution to $x = \cos(x)$.

Start w/ $f(x) = x - \cos(x)$ on $[0, \pi/2]$.

$f(0) = -1$. $f(\pi/2) = \pi/2$.

0 is between -1, & $\pi/2$

Try $m = \pi/4$.

$f(\frac{\pi}{4}) = \frac{\pi}{4} - \frac{\sqrt{2}}{2} \approx 0.078$

So $f(x) = 0$ has a

solution on $[0, \pi/4]$.

Repeat.

$m = \pi/8$.

$f(\pi/8) \approx -0.53$.

Solution of $f(x) = 0$ in $[\frac{\pi}{8}, \frac{\pi}{4}]$.