9/21
Clicker #1

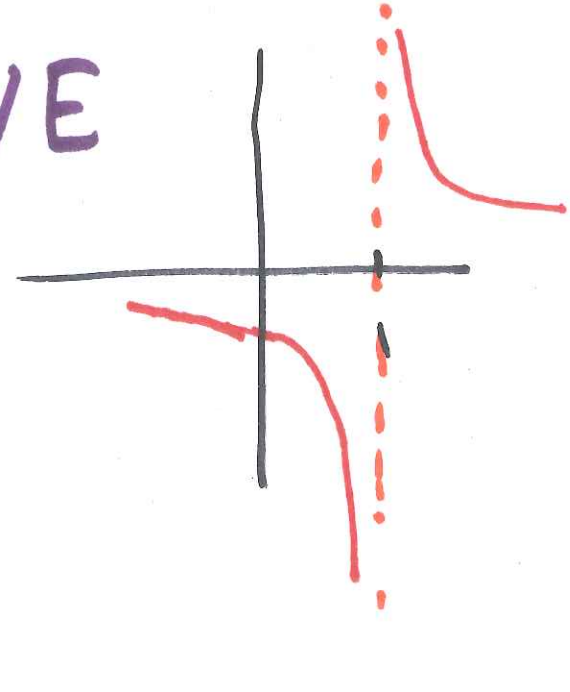
$$\lim_{x \to 1} \frac{x+1}{x^2-1}$$

$$\frac{x+1}{x^2-1} = \frac{x+1}{(x+1)(x-1)} = \frac{1}{x-1}$$

$$\lim_{x \to 1^+} \frac{1}{x-1} = +\infty$$

$$\lim_{x \to 1^-} \frac{1}{x-1} = -\infty \text{ . Then}$$

$$\lim_{x \to 1} \frac{1}{x-1} = \text{DNE}$$



$$\lim_{x \to 1} \frac{x-1}{x^2-1}$$

9/21
Chicken #1.

$$\frac{x-1}{x^2-1} = \frac{x-1}{(x-1)(x+1)} = \frac{1}{x+1}$$

$f(x) = 1/(x+1)$ is continuous at $x = 1$, so we may evaluate the limit by substitution –

$$\lim_{x \to 1} \frac{x-1}{x^2-1} = \lim_{x \to 1} \frac{1}{x+1}$$

$$= 1/2$$

A2.4-Continuity: Problem 14

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This set is

(1 point) **local/rmb-problems/piecewise-cont.pg**

Let f be defined by

$$f(x) = \begin{cases} x+5, & x \le 2 \\ Ax+B, & 2 < x < 7 \\ 2x+13 & 7 \le x \end{cases}$$

Find the values of A and B which make f continuous everywhere.

$A =$ $B =$ help

(numbers)

Solution: Recall f is continuous at a if f is defined near a and $\lim_{x \to a} f(x) = f(a)$.

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} Ax + B$$

$$= 2A + B.$$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} \cancel{x} + 5 = 7.$$

Want $\underline{2A + B = 7}$, so left + right limits agree

At ~~2~~ 7, need

$$\lim_{x \to 7^-} Ax + B = \lim_{x \to 7^+} 2x + 13$$

$$\underline{7A + B = 27.}$$

$$2A + B = 7$$

$$\underline{-\left(7A + B = 27 \right)}$$

$$-5A = -20$$

$$A = \underline{+4}$$

$$2 \cdot 4 + B = 7$$

$$B = -1.$$

$$\underline{A = 4, \ B = -1}$$

Check

$$f(2) = 7 = \lim_{x \to 2} f(x).$$

$$27 = f(7) = \lim_{x \to 7} f(x). \checkmark$$

A2.5-Evaluating-Limits: Problem 8

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This set is

(1 point)
Rogawski_Calculus_Early_Transcendentals_Second_Edition_Offering/2_Limi

Evaluate the limit:

$\lim_{x\to1} \frac{x^2-3x+2}{x^3-1} =$

Solution:

$\frac{x^2-3x+2}{x^3-1}$ is undefined

at $x=0$.

Simplify

$$\frac{x^2-3x+2}{x^3-1} = \frac{\cancel{(x-1)}(x-2)}{\cancel{(x-1)}(x^2+x+1)}.$$

$$\lim_{x\to1} \frac{x^2-3x+2}{x^3-1} = \lim_{x\to1} \frac{x-2}{x^2+x+1} = \frac{-1}{3}$$

A2.5-Evaluating-Limits: Problem 13

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This set is visib

(1 point) **local/rmb-problems/instant-vel.pg**
The position of a particle is given by the function $p(t) = 3t^3$.
Find the average velocity of the particle on the interval $[p, q]$.

The average velocity is . help

(formulas)

Hint:
Take the limit as q approaches p of the expression for the average velocity to find the instantaneous velocity at the time p.

The instantaneous velocity is . help

(formulas)

Solution:

Average velocity is

$$\frac{p(q)-p(p)}{q-p} = \frac{3q^3-3p^3}{q-p}$$

$$= \left(\frac{3\cancel{(p-q)}(p^2+pq+q^2)}{\cancel{q-p}}\right)$$

$$= \left(3(p^2+pq+q^2)\right).$$

$$\frac{3\cancel{(q-p)}(q^2+pq+p^2)}{\cancel{(q-p)}}$$

$$= 3(q^2+pq+p^2).$$

$$\lim_{q \to p} 3(q^2+pq+p^2) = \underline{9p^2}$$

A2.6-Trig-Limits: Problem 3

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This set is vi

(1 point)
Rogawski_Calculus_Early_Transcendentals_Second_Edition_Offering/2_Limits/

Evaluate using the Squeeze Theorem.

(Use symbolic notation and fractions where needed.)

$$\lim_{x\to\pi/2} (\cos x)\cdot\cos\left(\tan(8x)\right) =$$

help (fractions)

Note that

$$-1 \le \cos(\tan(\overset{8}{\curlyvee}x)) \le 1.$$

Thus

$$-|\cos(x)| \le \cos(x)\cos(\tan(\overset{8}{\curlyvee}x))$$

$$\le +|\cos(x)|$$

—New solution follows

We have

$$|\cos(y)| \le 1$$

for any y. Thus $|\cos(\tan(8x))| \le 1$. We also have $y \le |y|$.

Thus

$$\begin{aligned}
\cos(\tan(8x))\cos(x) &\le |\cos(\tan(8x))|\,|\cos(x)| \\
&\le |\cos(x)|.
\end{aligned}$$

Applying same argument to $-\cos(\tan(8x))\cos(x)$, we have

$$-\cos(\tan(8x))\cos(x)$$

$$\leq |-\cos(\tan(8x))| \cdot |\cos(x)|$$

$$\leq |\cos(x)|$$

so

$$\cos(\tan(8x))\cos(x)$$

$$\geq -|\cos(x)|.$$

Since we have

$$-|\cos(x)| \le \cos(x)\cos(\tan(8x)) \le |\cos(x)|.$$

and

$$\lim_{x\to\pi/2} \pm|\cos(x)| = 0.$$

The squeeze theorem tells us that

$$\lim_{x \to \pi/2} \cos(x)\cos(\tan(8x))$$

$$= 0. \qquad \square$$

Clicker question #3.

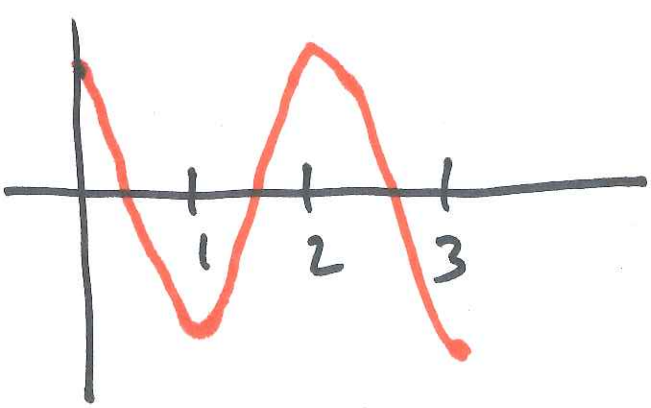
If $f(x)$ is continuous, and $f(x) = \cos(\pi x)$ for $x = 0, 1, 2, 3$, how many solutions are there of $f(x) = 0$?

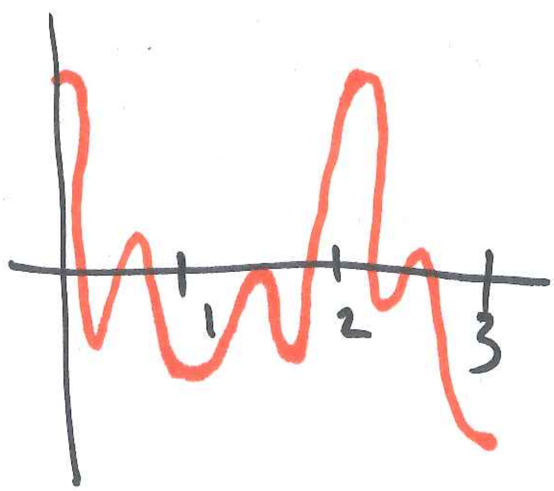
We have

$$f(0) = f(2) = +1$$

$$f(1) = f(3) = -1.$$

Thus the intermediate value theorem tells us that the equation $f(x)=0$ has at least one solution in each of the intervals $(0,1), (1,2), (2,3)$. However there may be many solutions. See below.





The correct answer is: I don't know.

A2.6-Trig-Limits: Problem 11

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(1 point)
Rogawski_Calculus_Early_Transcendentals_Second_Edition_Offering/

Evaluate the limit:

$\lim_{t \to 0} \frac{1-\cos 3t}{\sin 6t} =$

Solution:

We know

$$\lim_{t \to 0} \frac{\sin t}{t} = 1 \text{ and}$$

$$\lim_{t \to 0} \frac{1-\cos(t)}{t} = 0.$$

Then

$$\lim_{t \to 0} \frac{\sin(at)}{at} = 1, \text{ if } a \neq 0$$

and

$$\lim_{t \to 0} \frac{(1-\cos(at))}{at} = 0.$$

We can write

$$\frac{1-\cos(3t)}{\sin(6t)} = \frac{1-\cos(3t)}{t} \cdot \frac{t}{\sin(6t)}$$

$$= \frac{3}{6} \frac{1-\cos(3t)}{3t} \cdot \frac{1}{\frac{\sin(6t)}{6t}}$$

Then by the rules for limits of products and quotients,

$$\lim_{t \to 0} \frac{1-\cos(3t)}{\sin(6t)}$$

$$= \frac{1}{2} \lim_{t \to 0} \frac{1-\cos(3t)}{3t}$$

$$\times \lim_{t \to 0} \frac{1}{\frac{\sin(6t)}{6t}}$$

$$= \frac{1}{2} \cdot 0 = 0.$$

A2.8-IVT: Problem 5

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This set is visit

(1 point) **local/rmb-problems/ivt.pg**

Find an interval (a, b) of length 1 and so that we may use the values of the expression $xe^{\frac{x}{2}}$ at the endpoints of the interval and the intermediate value theorem to show that the equation

$$xe^{\frac{x}{2}} = 2$$

has a solution in the interval (a, b).

$a =$ 1 $b =$ 2

Solution:

If $f(x) = xe^{x/2}$, then $f(1) \approx 1.65$ and $f(2) \approx 5.44$, so there is a solution to $f(x) = 2$ on $(1, 2)$.