

①

Lee. 12.

9/23/15.

The derivative f' .

Suppose f is a function defined in an open interval containing a . We call

$$\lim_{x \rightarrow a} \frac{f(x) - f(a)}{x - a} = f'(a).$$

the derivative of f at a , provided the limit exists. We write $f'(a)$ for the derivative.

If $f'(a)$ exists, we say that f is differentiable at a .

②

Alternate definition

$$\lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = f'(a).$$

why? two definitions?

Example

$$f(x) = mx + b.$$

what is $f'(7)$?

Answer m .

$$\cancel{mx+b} - (\cancel{mx} + b)$$

$$\frac{mx+b - (m \cdot 7 + b)}{(x-7)}.$$

$$= \frac{\cancel{m(x-7)}}{\cancel{x-7}} = m.$$

$$\lim_{x \rightarrow m} m = m.$$

Example $f(x) = x^n$,
 $n = 1, 2, \dots$, find $f'(a)$.

Solution.

$$\frac{x^n - a^n}{x - a}$$

$$\text{or } \frac{(a+h)^n - a^n}{h}$$

$$= \frac{(x-a)(x^{n-1} + xa^{n-1} + \dots + a^{n-1})}{(x-a)}$$

$$= \frac{(x-a)(x^{n-1} + ax^{n-2} + \dots + a^{n-1})}{(x-a)}$$

$$\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a}$$

$$= \lim_{x \rightarrow a} (x^{n-1} + ax^{n-2} + \dots + a^{n-1}).$$

$$= \frac{n a^{n-1}}{1}$$

$$f(x) = x^n$$

$$f'(a) = n a^{n-1}$$

Try to take the limit

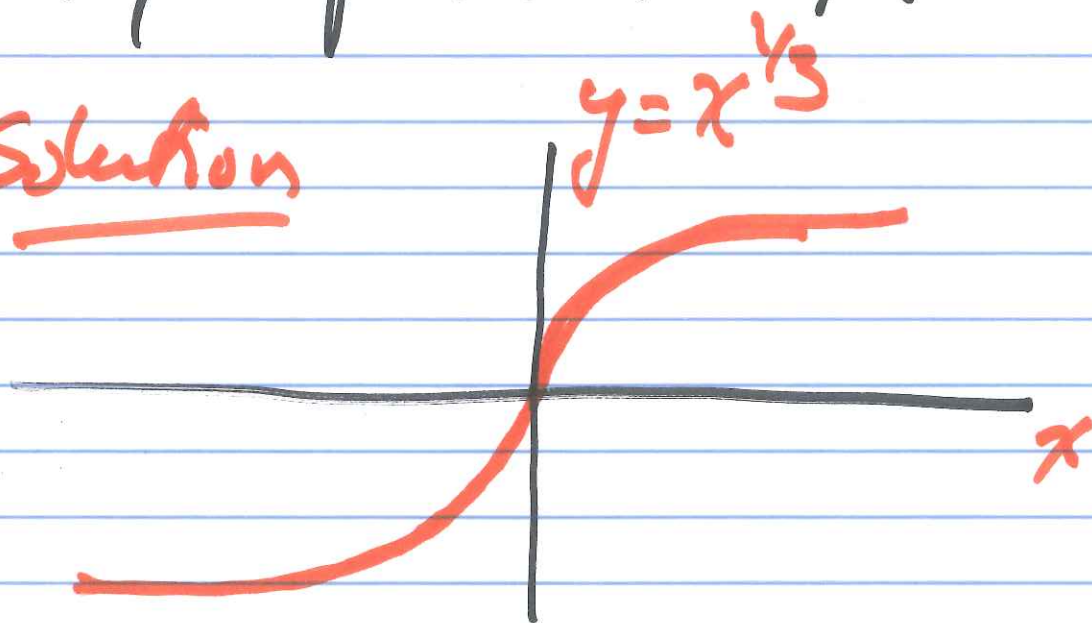
$$\lim_{h \rightarrow 0} \frac{(a+h)^n - a^n}{h}$$

Not differentiable?

Example Let $f(x)$
 $= \sqrt[3]{x} = x^{1/3}$

Try to find $f'(0)$.

Solution



6

$$\frac{f(x) - f(0)}{x - 0} = \frac{x^{1/3} - 0}{x - 0}$$

$$= x^{-2/3}$$

$$\lim_{x \rightarrow 0} x^{-2/3} = +\infty$$

limit does not exist
 f is not differentiable
at 0.

Approximating $f'(a)$.

$$f'(a) = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

We expect

$$\frac{f(a+h) - f(a)}{h}$$
 is

close to $f'(a)$
 when h is small.

Example 1) $f(x) = \sin(x)$
 approximate $f'(0)$.

8

Solution

$$\frac{f(h+0) - f(0)}{h} = \frac{\sin(h)}{h}$$

Approximate

<u>h</u>	<u>$\sin(h)/h$</u>
0.1	0.9983
0.02	0.9993...

(9)

Addendum

$$(x-a)(x^{n-1} + ax^{n-2} + \dots + a^{n-1})$$

$$= x(x^{n-1} + ax^{n-2} + \dots + a^{n-1}) - a(x^{n-1} + ax^{n-2} + \dots + a^{n-1})$$

$$= x^n + ax^{n-1} + \dots + xa^{n-1} - ax^{n-1} - \dots - xa^{n-1} - a^n$$

$$= \underline{x^n - a^n}$$