9/25/15

Recall if $f(x) = x^n$,
then $f'(x) = nx^{n-1}$!

- Proven for $n = 1, 2, \ldots$
- True for all n. We will use it now & prove it later.

Leibniz Notation.

$$\frac{df}{dx} = f' \quad (= \dot{f})$$

why? Δ = change

$$\Delta f = f(x+h) - f(x).$$

$$\Delta x = (x+h) - x.$$

$$\lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}$$

$$= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= f' \text{ or } \frac{df}{dx}(x)$$

Mathematicians prefer $f'(x)$, since it is easier to show the independent variable.

If $f(x) = b^x$.

$$f'(x) = \lim_{h \to 0} \frac{b^{x+h} - b^x}{h}$$

$$= \lim_{h \to 0} b^x \frac{(b^h - 1)}{h}$$

$$= b^x \lim_{h \to 0} \frac{b^h - 1}{h}$$

$$= m(b) \cdot b^x$$

why is e^x special?

$m(e) = 1$.

$$\boxed{\frac{d}{dx} e^x = e^x}$$

New derivatives from old.

Suppose f & g are differentiable, then

$f+g$ is differentiable

and $(f+g)' = f' + g'$.

~~Also~~ Also cf is differentiable

$(cf)' = cf'$, where c

is a constant.

Why? Limit laws.

$$(f+g)'(x)$$

$$= \lim_{h\to 0} \frac{(f(x+h)+g(x+h)) - (f(x)+g(x))}{h}$$

$$= \lim \left(\frac{f(x+h)-f(x)}{h} + \frac{g(x+h)-g(x)}{h} \right)$$

$$= \lim_{h\to 0} \left(\frac{f(x+h)-f(x)}{h} \right) + \lim \left(\frac{g(x+h)-g(x)}{h} \right)$$

$$= f'(x) + g'(x)$$

<u>Example</u> Find

$$\frac{d}{dx}(3x^4 + 4x^3)$$

$$= \frac{d}{dx}(3x^4) + \frac{d}{dx}(4x^3)$$

$$= 3\frac{d}{dx}x^4 + 4\frac{d}{dx}x^3$$

$$= 3 \cdot 4x^3 + 4 \cdot 3x^2$$

$$= \underline{12x^3 + 12x^2.}$$

Example (Clicker).

~~Ex~~ $f(x) = (3x)^2 + 2e^x$.

Find $f'(x)$.

$$f(x) = (3x)^2 + 2e^x$$

$$= 9x^2 + 2e^x$$

$$f'(x) = 18x + 2e^x$$

Differentiability & Continuity.

<u>Theorem.</u> If f is differentiable at x, then f is continuous at x.

why? Need

$$\lim_{x\to a} f(x) \stackrel{?}{=} f(a) \quad \text{or}$$

$$\lim_{x\to a} \left(f(x) - f(a)\right) \stackrel{?}{=} 0$$

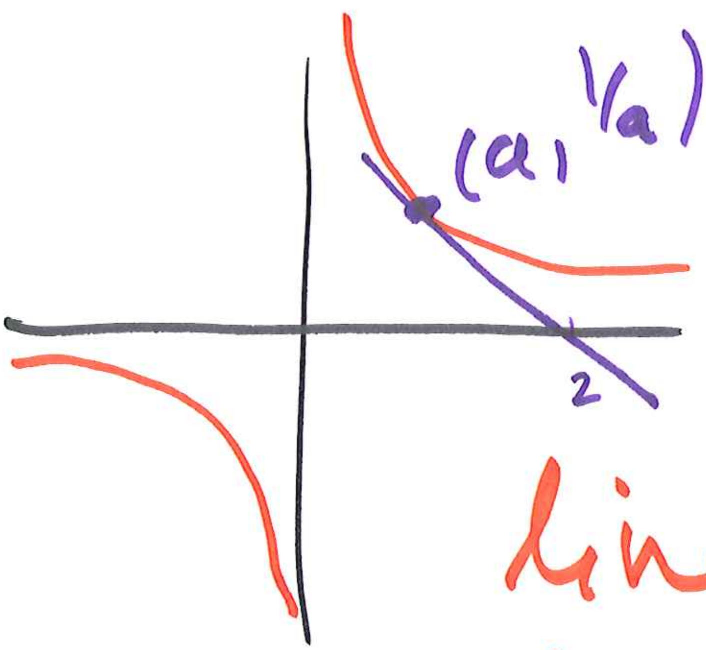
$$\lim_{x\to a} \left(\frac{f(x)-f(a)}{x-a}(x-a)\right)$$

$$= \lim_{x \to a} \frac{(f(x) - f(a))}{x - a} \cdot \lim_{x \to a} (x - a)$$

$$\cancel{= f'(a) \cdot 0 = 0}$$

$$= f'(a) \cdot 0 = 0$$

Let $f(x) = \frac{1}{x}$. Find a so that the tangent line at $x = a$ passes thru $(2, 0)$



$f(x) = \frac{1}{x} = x^{-1}$

$f'(x) = -x^{-2}$.

Tangent line at a is

$$y - \frac{1}{a} = \frac{-1}{a^2}(x - a).$$

Wan this line to pass thru $(x,y) = (2,0)$.

$$0 - \frac{1}{a} = -\frac{1}{a^2}(2-a).$$

Solve for a

$$-\frac{1}{a} = -\frac{2}{a^2} + \frac{1}{a}$$

$$a^2 \frac{2}{a^2} = \frac{2}{a} \cdot a^2$$

$2 = 2a$ or $\underline{a = 1}$.