

Product rule.

9/28/15

Theorem. If f and g are differentiable at a , then $f \cdot g$ is differentiable at a and

$$(fg)'(a) = f'(a)g(a) + f(a)g'(a).$$

Why?

$$-f(x)g(x+h) + f(x)g(x+h)$$

Consider $\forall$

$$\frac{f(x+h)g(x+h) - f(x)g(x)}{h}$$

$$= g(x+h) \left(\frac{f(x+h) - f(x)}{h} \right) + f(x) \left(\frac{g(x+h) - g(x)}{h} \right)$$

Oops Set $x=a$. Take limits
as $h \rightarrow 0$

Use that g is continuous 2
so $\lim_{h \rightarrow 0} g(a+h) = g(a)$.

Obtain

$$(fg)'(a) = g(a)f'(a) + f(a)g'(a). \quad \square$$

Example

$$\frac{d}{dx}(x^2 e^x).$$

$$\frac{d}{dx}(x^2 e^x) = \cancel{e^x} \frac{d}{dx} \cancel{e^x}$$

$$= e^x \frac{d}{dx} x^2 + x^2 \frac{d}{dx} e^x$$

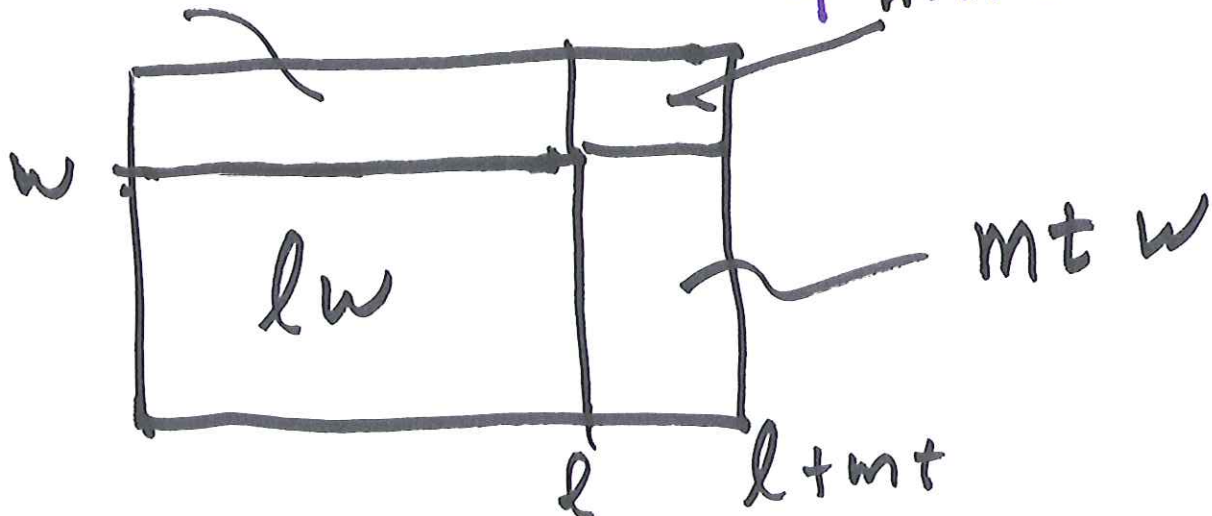
$$= 2x e^x + x^2 e^x.$$

~~Let~~ Consider a rectangle w/ sides of length
 $l(t) = l + mt$ and
 width $w(t) = w + nt$
 Then the area is

$$A(t) = l(t) \cdot w(t)$$

$$= (l + mt)(w + nt)$$

$$= lw + (mw + ln)t + mnt^2$$



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Find

$$\frac{d}{dx} ((1+2x)^2 e^x)$$

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$$= \frac{d}{dx} (1 + 4x + 4x^2) e^x$$

$$= (4 + 8x) e^x + (1 + 4x + 4x^2) e^x$$

$$= (5 + 12x + 4x^2) e^x$$

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Derivative of a
quotient $\frac{f}{g}$.

Write $\frac{f}{g} = f \cdot \left(\frac{1}{g}\right)$.

only need to consider
reciprocals:

$$\left(\frac{1}{g}\right)'$$

Reciprocal rule -

If g is differentiable at a
and $g(a) \neq 0$, then

$$\left(\frac{1}{g}\right)'(a) = \frac{-g'(a)}{g(a)^2}.$$

Find the tangent line
to $f(x) = \frac{1}{1+x^2}$ at $\underline{x=1}$.

Solution. $f(x) = \frac{1}{1+x^2}$

$$f'(x) = \frac{-2x}{(1+x^2)^2}$$

$$f'(1) = \frac{-2}{4} = -\frac{1}{2}.$$

Tangent line passes thru

$$(1, \frac{1}{2}) = (1, f(1)). \quad \text{Tangent}$$

line is

$$y - \frac{1}{2} = -\frac{1}{2}(x-1)$$

Simplify $y = -\frac{1}{2}x + 1.$

Power rule for $n = -1, -2, \dots$ 7

$$\begin{aligned}\frac{d}{dx} x^{-n} &= \frac{d}{dx} \frac{1}{x^n} \\ &= \frac{-n x^{n-1}}{x^{2n}} \\ &= -n x^{-n-1}.\end{aligned}$$

Quotient rule -

~~If $\frac{d}{dx}$~~

Quotient rule.

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If f, g are differentiable at a , and $g(a) \neq 0$, then

$$\left(\frac{f}{g}\right)'(a) = \frac{f'(a)g(a) - g'(a)f(a)}{g(a)^2}.$$

Example Find $f'(x)$ for

$$f(x) = \frac{1-x^2}{1+x^2}.$$

$$\begin{aligned} f'(x) &= \frac{(1-x^2)'(1+x^2) - (1-x^2)(1+x^2)'}{(1+x^2)^2} \\ &= \frac{-2x(1+x^2) - (1-x^2)(2x)}{(1+x^2)^2} \end{aligned}$$

$$= \frac{-2x - 2x^3 - 2x + 2x^3}{(1+x^2)^2}$$

$$= \frac{-4x}{(1+x^2)^2}$$