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If $f(x) = x^3$, then

$f^{(3)}(x) =$

A 6 D $27x^6$

B x^9

C x^6

Higher order derivatives

f function

f' is first derivative

f'' is 2^{nd} derivative

$$f'' = (f')'$$

<u>Example.</u> If $f(x) = x^2$,
$f'(x) = 2x$, $f''(x) = 2$.

f''' is 3rd derivative

Alternate notation
$f^{(n)}$ ~~is~~ is the n^{th} derivative.

e.g. $f' = f^{(1)}$, $f'' = f^{(2)}$

$$f^{(n-1)\prime} = f^{(n)}.$$

Leibniz form.

$$f^{(n)} = \frac{d^n f}{dx^n}$$

<u>Example</u>. Let $f(x) = e^x$

find $f^{(2015)}(x)$.

$f(x) = e^x$, $f'(x) = e^x$, $f''(x) = e^x$

$\ldots$ $f^{(2015)}(x) = e^x$.

<u>Example</u>. If $f(x) = x^2 e^x$,

find $f''(x)$.

$$f'(x) = (x^2)' \cdot e^x + x^2 e^x$$

$$= 2x e^x + x^2 e^x$$

$$= (x^2 + 2x) e^x.$$

$$f''(x) = ((x^2+2x) e^x)'$$

$$= (2x+2) e^x + (x^2+2x) e^x$$

$$= (x^2 + 4x + 2) e^x.$$

If $f(x) = xe^x$, then

$f^{(100)}(x) =$

A $x^{100}e^x$

B $100xe^x$

C $(x+100)e^x$

D e^x.

$f(x) = xe^x$, $f'(x) = (x)'e^x + x(e^x)'$

$= 1e^x + xe^x$

$= (x+1)e^x$

$f''(x) = 1 \cdot e^x + (x+1)e^x = (x+2)e^x$.

<u>Example</u> Find a polynomial $p(x) = ax^2 + bx + c$

so that $\cancel{p(0) = e^0}$ if

$\cancel{p'(0) =}$

$f(x) = e^x$, then

$p(0) = f(0)$, $p'(0) = f'(0)$

$p''(0) = f''(0)$.

Compare values of $p(x)$ to e^x.

$$p(x) = ax^2 + bx + c$$

$$p(0) = c. = e^0 = 1$$

$$p'(x) = 2ax + b$$

$$p'(0) = b = f'(0) = 1$$

$$p''(x) = 2a,$$

$$p''(0) = 2a = 1.$$

$$c = 1, \quad b = 1, \quad a = 1/2$$

$$p(x) = \frac{1}{2}x^2 + x + 1.$$

x	e^x	$p(x)$
0	1	1
0.01	1.0100501 67...	1.01005
0.2	1.22140...	1.22.

Trig functions.

$f(x) = \sin(x)$, $f'(x) =$ __

Recall -

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$$

$$\lim_{x \to 0} \frac{\cos(x) - 1}{x} = 0$$

$$\sin(x+y) = \sin(x)\cos(y) + \cos(x)\sin(y).$$

$f'(x) = \sin'(x) =$

$$\sin'(x) = \lim_{h \to 0} \left(\frac{\sin(x+h) - \sin(x)}{h} \right).$$ 8

Simplify difference quotient

$$\frac{\sin(x+h) - \sin(x)}{h}$$

$$= \frac{\sin(x)\cos(h) + \cos(x)\sin(h) - \sin(x)}{h}$$

$$= \cos(x) \cdot \frac{\sin(h)}{h} + \sin(x)\left(\frac{\cos(h)-1}{h}\right)$$

$$\lim_{h \to 0} (\quad) = \cos(x) \cdot 1 + \sin(x) \cdot 0$$

-or-

$$\boxed{\sin'(x) = \cos(x)}$$

$$\boxed{\cos'(x) = -\sin(x)}$$

Example $\frac{d}{dy} \tan(x).$

Solution

$$\tan'(x) = \left(\frac{\sin(x)}{\cos(x)}\right)'$$

$$= \frac{\sin'(x)\cos(x) - \cos'(x) \cdot \sin(x)}{(\cos(x))^2}$$

$$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)}.$$

$$= \frac{1}{\cos^2(x)} \quad \text{Pyth. id.}$$

$$= \sec^2(x).$$

$$\boxed{\frac{d}{dx}\tan(x) = \sec^2(x).}$$

$$\boxed{\frac{d}{dx}\cot(x) = -\csc^2(x)}$$

$$\boxed{\frac{d}{dx}\sec(x) = \sec(x)\tan(x)}$$

$$\boxed{\frac{d}{dx}\csc(x) = -\csc(x)\cot(x)}$$

<u>Clicker</u>

$f(x) = \sin(x)\cos(x).$

$f'(x) = \sin'(x)\cos(x)$

$+ \sin(x)\cos'(x).$

$= \cos^2(x) - \sin^2(x).$

$(= \cos(2x).)$, by

double-angle formula.