If $f(x) = \cos(x)\sec(x)$, find $f'(x)$. (Think!)

$$f(x) = \cos(x) \cdot \frac{1}{\cos(x)}$$

$$= 1$$

$$f'(x) = 0$$

If $f(x) = \cos(x) + \sin(x)$, find $f^{(401)}(x)$.

$f^{(5)} = f(x) = \cos(x)$. $\quad f'''(x) = \sin(x)$.

$f^{(6)} = f'(x) = -\sin(x)$ $\quad f^{(4)}(x) = \cos(x)$.

$f''(x) = -\cos(x)$ $\quad f^{(8)} = f^{(12)}$

$= f^{(16)} \ldots$

Let $g(x) = \cos(x)$. We compute $g', g'' \ldots$ and look for a pattern.

$$g(x) = \cos(x). \qquad = g^{(4)}(x)$$

$$g'(x) = -\sin(x) \qquad = g^{(5)}(x)$$

$$\vdots$$

$$g''(x) = -\cos(x) \qquad = g^{(6)}(x)$$

$$g'''(x) = \sin(x) \qquad = g^{(7)}(x)$$

$$g^{(4)}(x) = \cos(x) = g(x)$$

Since $\cos^{(4k)}(x) = \cos(x)$

for $k = 1, 2, \ldots$

We have

$$\frac{d^{401}}{dx^{401}} \cos(x) = \frac{d}{dx} \cos(x)$$

$$= -\sin(x).$$

Add

$$\frac{d^{401}}{dx^{401}} \sin(x) = \frac{d}{dx} \sin(x)$$

$$= \cos(x)$$

Then if $f(x) = \cos(x) + \sin(x)$

$f^{(401)}(x)$

If $f(x) = \dfrac{\cos(x)}{2+\sin(x)}$,
find the tangent line at 0.

Tangent line at 0 passes thru $(0, f(0)) = (0, 1/2)$.

Slope is $f'(0)$

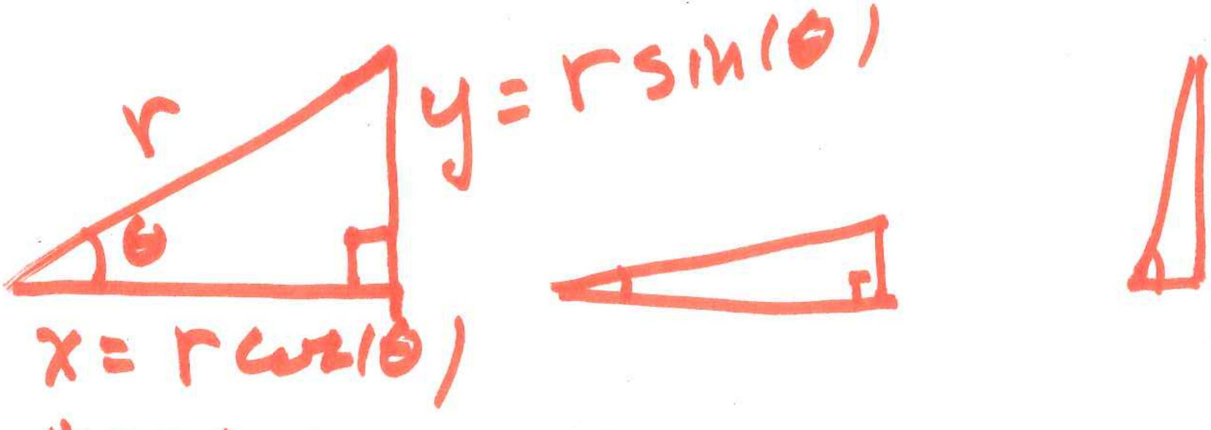
$$f'(x) = \frac{-(2+\sin(x))\sin(x) - \cos(x)\,(\cos(x))}{(2+\sin(x))^2}$$

$$f'(0) = \frac{0 - 1}{4} = -1/4.$$

$$y - \tfrac{1}{2} = -\tfrac{1}{4}(x - 0), \quad \cancel{y = -\tfrac{1}{4}x + \tfrac{3}{4}}$$

$$y = -\tfrac{1}{4}x + \tfrac{1}{2}$$

Let T be a right triangle with hypotenuse r and acute angle θ. For which value of θ is the rate of change of area 0?



$\cancel{A(\theta) =}$

$$A(\theta) = \frac{1}{2} r^2 \sin(\theta) \cdot \cos(\theta)$$

$$A'(\theta) = \frac{r^2}{2}\left(\cos^2(\theta) - \sin^2(\theta)\right).$$

$A'(\theta) = 0$ if

$\cos^2(\theta) = \sin^2(\theta)$

or $\cos(\theta) = \pm\sin(\theta)$

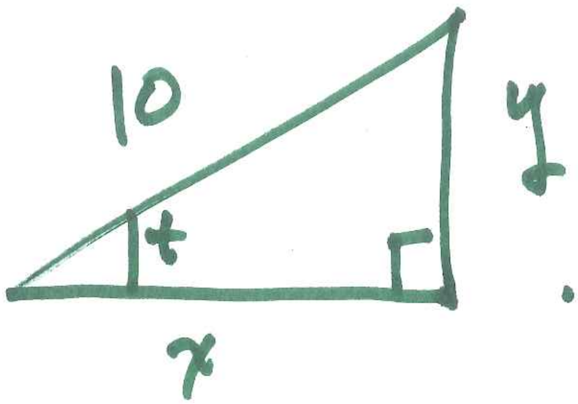
$\theta = \underline{\underline{\pi/4}}, \frac{3\pi}{4}, \frac{5\pi}{4} \ldots$

Only $\pi/4$ is possible in a right triangle.

$\underline{\theta = \pi/4}$

For θ small, A is small & when θ is close to $\pi/2$, A is again small. Somewhere in between, we expect A to quit increasing & start decreasing.

It is reasonable to expect this transition to take place at $\theta = \pi/4$:



Find $\frac{dy}{dt}$ @ $t = \pi/6$.

$$y = 10\sin(t)$$

$$\frac{dy}{dt} = 10\cos(t).$$

$t = \pi/6$ $$\frac{dy}{dt} = 10\cos(\pi/6)$$

$$= 10 \cdot \frac{\sqrt{3}}{2} = 5\sqrt{3}$$

Chain rule

Know $f'(x)$, $g'(x)$, can we find $(f \circ g)' = \frac{d}{dx}(f(g(x)))$?

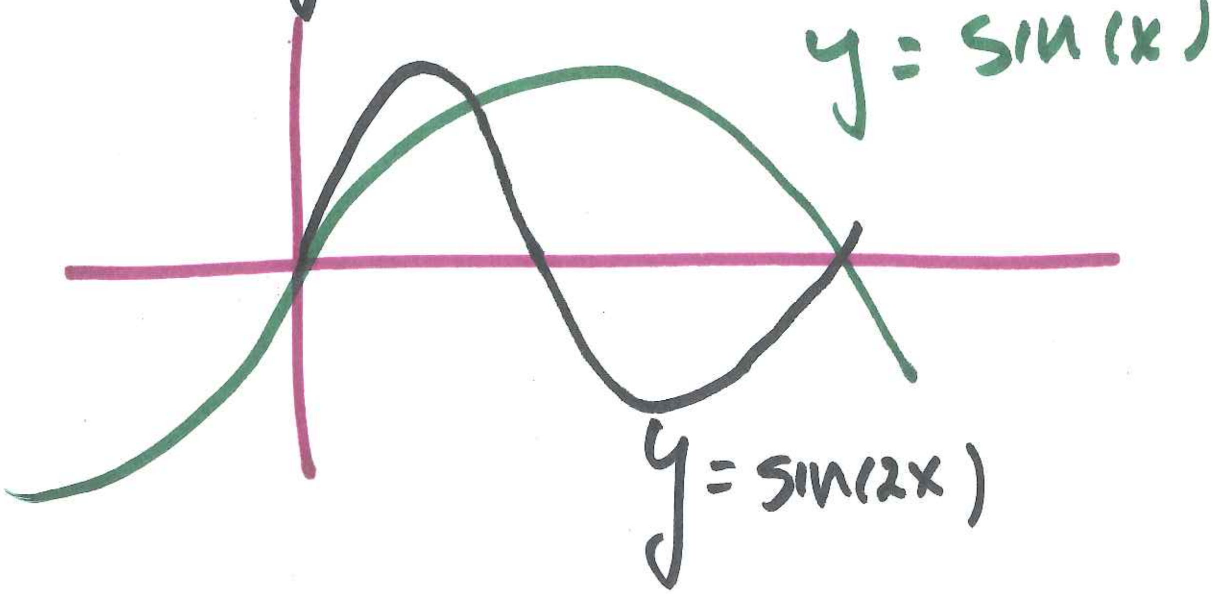
Yes.

Example Graph

$f(x) = \sin(x)$ and

$h(x) = \sin(2x) = (f \circ g)(x)$

w/ $g(x) = 2x$.



Slope at origin of black curve is 2 times the slope of the green curve.

Claim. If g is differentiable at x and f is differentiable at $g(x)$, then $(f \circ g)$ is differentiable at x and

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x).$$

Why? Almost a proof -

$$\lim_{h \to 0} \frac{f(g(x+h)) - f(g(x))}{h} \cdot \frac{g(x+h)-g(x)}{g(x+h)-g(x)}$$

$$= \lim_{h \to 0} \frac{f(g(x+h)) - f(g(x))}{g(x+h)-g(x)} \cdot \frac{g(x+h)-g(x)}{h}$$

$$\downarrow \qquad\qquad\qquad \downarrow$$

$$f'(g(x)) \cdot g'(x).$$

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$$\sin^2(x+1) = (\sin(x+1))^2$$

$$= \sin^2(x+1).$$

A or C