10/19/15

B3.7 #8

Review 2.2

Calculate the following derivative if

$f(x) = e^{-1x}$

$g(x) = \underline{x} + x^{-12}$

(Use symbolic notation and fractions where needed.)

$\frac{d}{dx} f(g(x)) =$ help (fractions)

$$\frac{d}{dx}(f(g(x))) = \frac{df}{dx}(g(x))\,\frac{dg}{dx}(x).$$

$$= f'(g(x)) \cdot g'(x).$$

$f'(x) = -e^{-x}$

$g'(x) = 1 - 12x^{-13}$

$$= -e^{-(x+x^{-12})} \cdot (1 - 12x^{-13}).$$

B3-8-3.9 #3

(2 points)
Rogawski_Calculus_Early_Transcendentals_Second_Edition_Offering/3_Differentiation/3.8_Deriva

Let $g(x)$ be the inverse of $f(x) = x^3 + 2x + 6$. Calculate $g(9)$ [**without finding a formula for** $g(x)$] and then calculate $g'(9)$.

$g(9)$ = 1

$g'(9)$ = 1/5

~~g(x)~~ $f(x) = x^3 + 2x + 6$. Let $g(x) = f^{-1}(x)$. Find $g(9)$ & $g'(9)$.

Try to solve

$x^3 + 2x + 6 = 9$ ☹

Compute ~~$f(0) = 6$~~ ✗

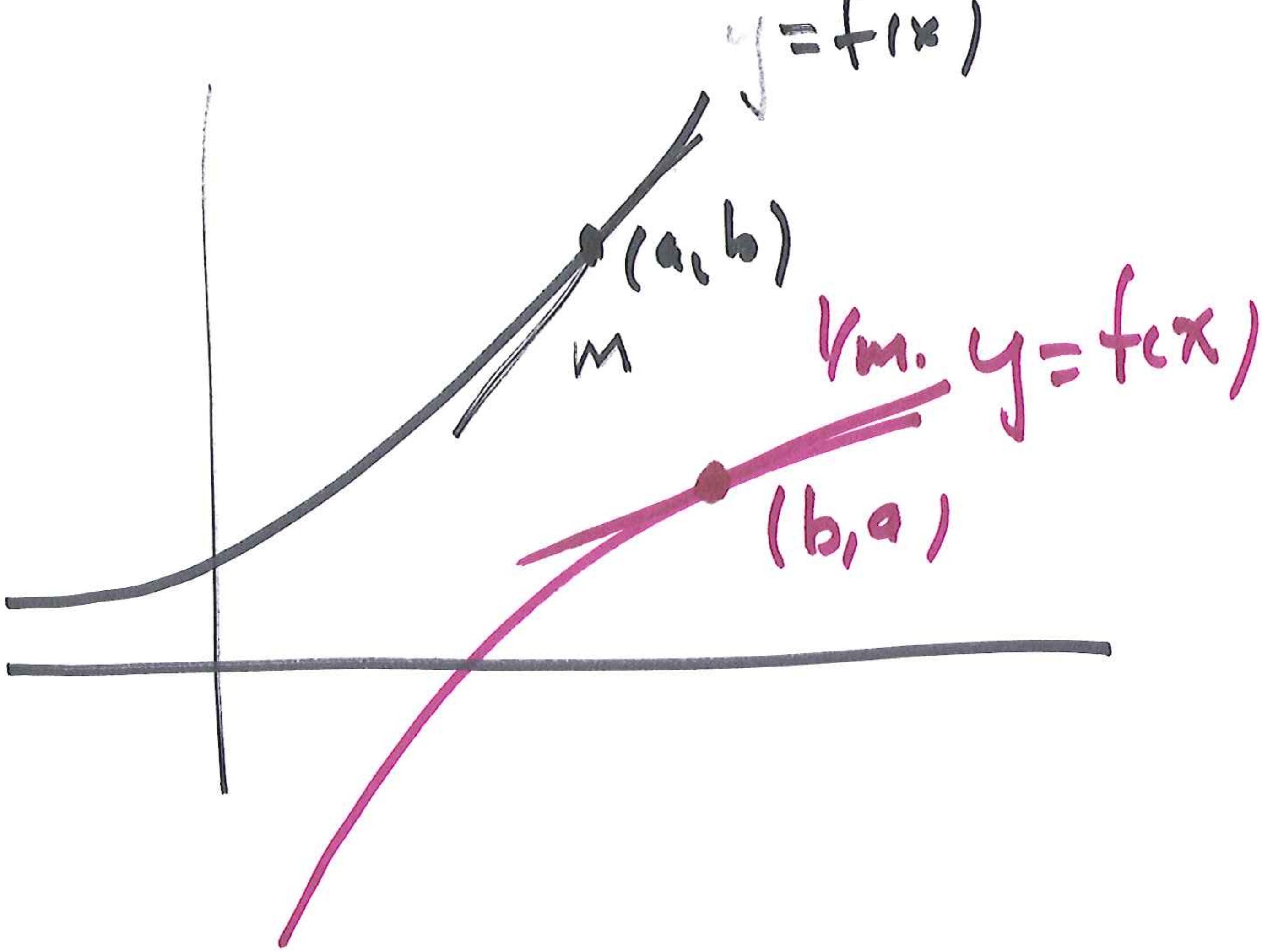
$f(1) = 9$ ✓.

So $f^{-1}(9) = 1. = g(9).$

Formula for g'

$$g'(9) = \frac{1}{f'(1)} = \frac{1}{5}.$$

$$f'(x) = 3x^2+2, \; f'(1) = 5$$



Clicker. $f(1) = 2.$

$f'(1) = 3, \quad f'(2) = 4.$

Let $g = f^{-1}$

$$g'(2) = \frac{1}{f'(1)} = \frac{1}{3}$$

B3.10 #9

(1 point)
Rogawski_Calculus_Early_Transcendentals_Second_Edition_(

Find an equation of the tangent line at the given point.

$e^{5x-y} = \frac{5x}{y}$, (2,10).

$y - 10 =$

Solution:

$$\frac{d}{dx} e^{5x-y} = \frac{d}{dx} \frac{5x}{y} = \frac{d}{dx}(5xy^{-1}).$$

$$\left(5 - \frac{dy}{dx}\right) e^{5x-y} = 5y^{-1} - 1(5x)y^{-2}\frac{dy}{dx}.$$

Subsntute $x=2, y=10$

$$\left(5 - \frac{dy}{dx}\right) e^{0} = \frac{5}{10} - 1\,\frac{10}{100}\,\frac{dy}{dx}$$

$$5 - \frac{dy}{dx} = \frac{1}{2} - \frac{1}{10}\frac{dy}{dx}$$

$$\frac{9}{2} = \frac{9}{10}\frac{dy}{dx}$$

$$\frac{dy}{dx} = \frac{10}{9} \cdot \frac{9}{2} = 5.$$

Line thru $(x, y) = (2, 10)$ w/ slope is 5.

$$y - 10 = 5(x-2).$$

No testing in first 4 rows.

B3.11 #1

At what rate is water pouring into a rectangular bathtub with base $66\ \mathrm{ft}^2$ if the water level rises at a rate of 1.4 ft/min.

(Give your answer as an exact number.)

Rate is = $\mathrm{ft}^3/\mathrm{min}$ help (numbers)

$$V = 66h.$$

$$\frac{dv}{dt} = 66 \cdot \frac{dh}{dt}$$

$$= 66\ ft^2 \cdot 1.4\ \frac{ft}{min}$$

$$= 66 \cdot 1.4\ ft^3/min.$$

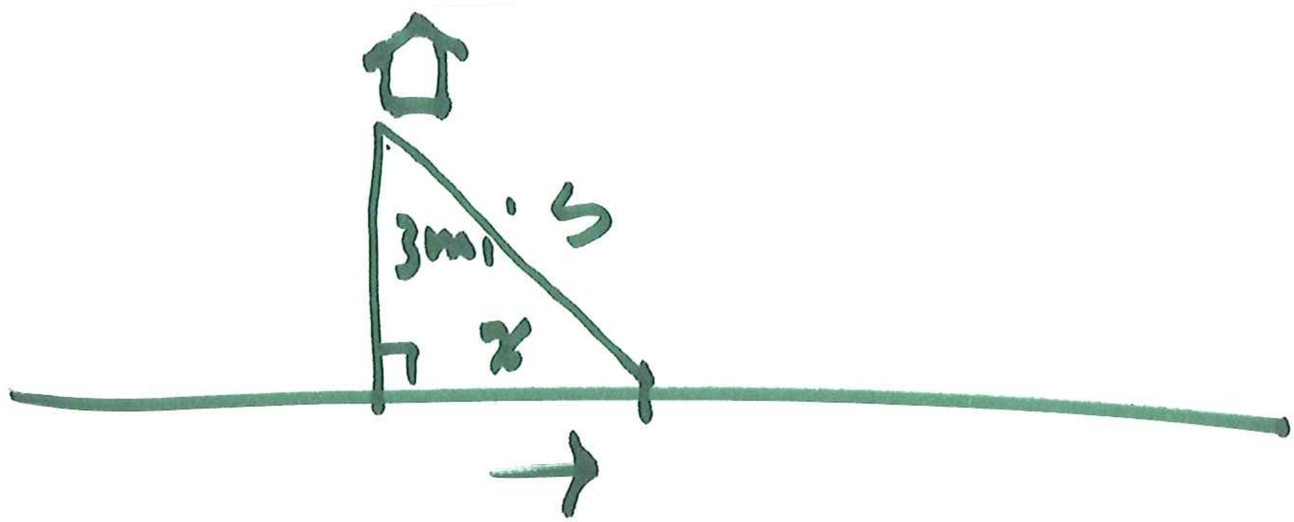
B3.11 #6

A road perpendicular to a highway leads to a farmhouse located 3 mile away. (See Figure 9, page 204 of the text.) An automobile traveling on the highway passes through this intersection at a speed of 50mph.

How fast is the distance between the automobile and the farmhouse increasing when the automobile is 9 miles past the intersection of the highway and the road?

The distance between the automobile and the farmhouse is increasing at a rate of

miles per hour.



$v = 50$ m/h.

Find $\frac{ds}{dt}$ when $x = \cancel{1}.\,9.$

$s^2 = x^2 + 9 \qquad s = \sqrt{x^2 + 9}$

$$2s\frac{ds}{dt} = 2x\frac{dx}{dt}$$

$$\frac{ds}{dt} = \frac{x}{s}\frac{dx}{dt}. \qquad x = 9.$$

$$s = \sqrt{81+9}$$

$$= \frac{9}{3\sqrt{10}} \cdot 50 \qquad = \sqrt{90}$$

$$= 3\sqrt{10}$$

$$= \frac{150}{\sqrt{10}}$$

$$= \frac{15 \cdot 10}{\sqrt{10}} = 15 \cdot \sqrt{10} \ \frac{\text{miles}}{\text{hour}}$$

B3.11 #2

The radius of a circular oil slick expands at a rate of 7 m/min.

(a) How fast is the area of the oil slick increasing when the radius is 20 m?

$\frac{dA}{dt} =$ m^2/min

(b) If the radius is 0 at time $t = 0$, how fast is the area increasing after 2 mins?

$\frac{dA}{dt} =$ m^2/min

$$A = \pi r^2$$

$$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$$

$$= 2\pi r \cdot 7 m/min.$$

a) $r = 20$ m

$$= 280\pi \; m^2/min.$$

(b) After 2 minutes

$r = 14$ meters

$$\frac{dA}{dt} = 2\pi r \cdot 7 \text{ m/min}$$

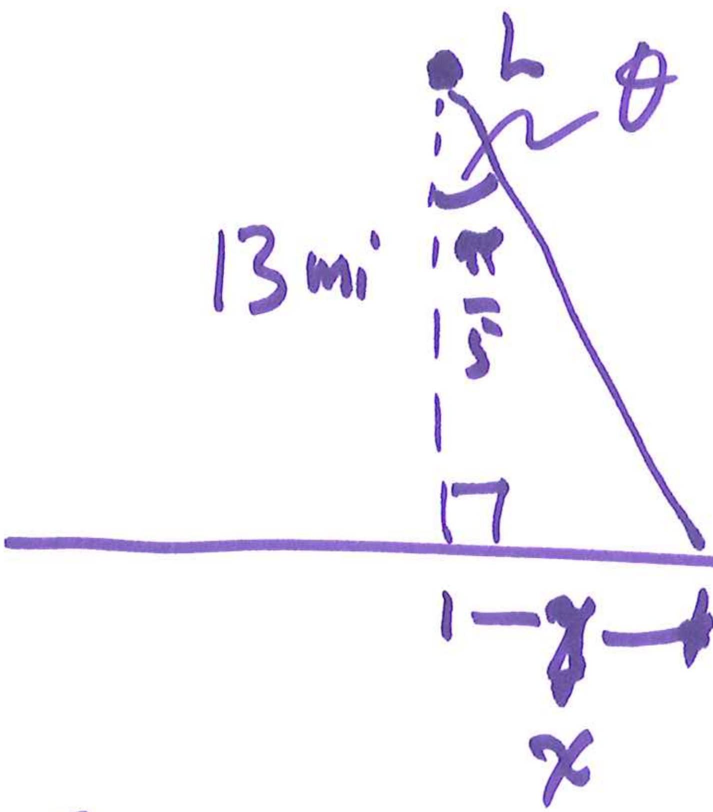
$$= 28\pi \cdot 7 \text{ m}^2/\text{min.}$$

$$= 196\pi \text{ m}^2/\text{min.}$$

B3.11 #9

A searchlight rotates at a rate of 3 revolutions per minute. The beam hits a wall located 13 miles away and produces a dot of light that moves horizontally along the wall. How fast (in miles per hour) is this dot moving when the angle θ between the beam and the line through the searchlight perpendicular to the wall is $\frac{\pi}{5}$? Note that $d\theta/dt = 3(2\pi) = 6\pi$. rad/min

Speed of dot = mph.



Find $\frac{dx}{dt}$.

when $\theta = \frac{\pi}{5}$.

Given $\frac{d\theta}{dt} = 360\pi$ radians/hour

$$\tan(\theta) = \frac{x}{13}$$

$$x = 13\tan(\theta).$$

$$\frac{dx}{dt} = 13\sec^2(\theta)\cdot\frac{d\theta}{dt}$$

$$= \frac{13\text{ mi}}{\cos^2\left(\frac{\pi}{5}\right)} \cdot 360\pi\ \frac{\text{rad}}{\text{hour}}$$