L25/10/23/15

If f is continuous on a closed bounded interval $[a,b]$, then f has an absolute maximum value on $[a,b]$. { For which function does this theorem apply?

(a) $\tan(x)$ on $[0, \frac{\pi}{4}]$

(b) $\frac{1}{x}$ on $[1, \infty)$. Not bounded.

(c) $\sin(x)$ on $(0,1)$ Not closed.

(d) $\tan(x)$ on $[\frac{\pi}{4}, \frac{3\pi}{4}]$ Not continuous

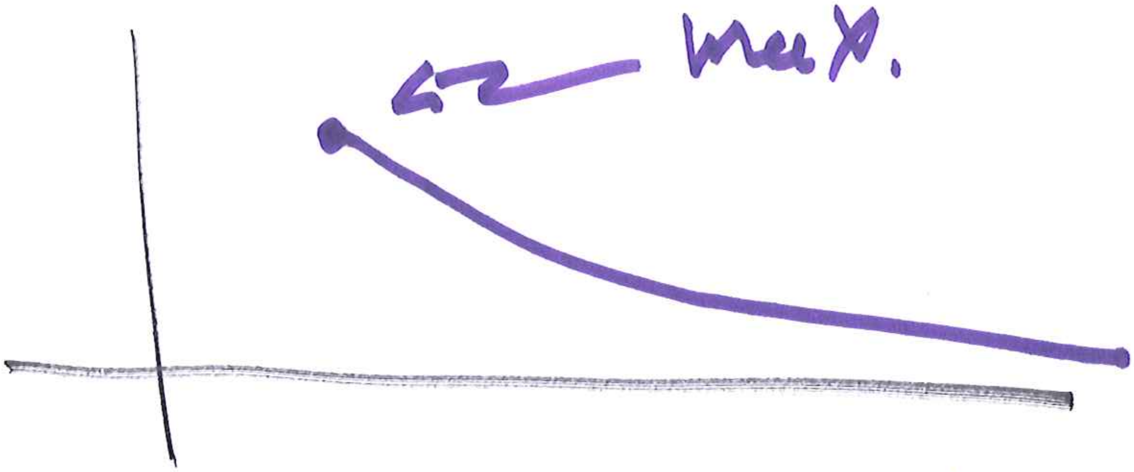
Def. If f is a function on an interval I, we say $f(a)$ is an absolute maximum ~~for~~ value for f on I if $f(x) \le f(a)$ for all x in I.

Exercise: Define absolute minimum value.

- Recall theorem from the clicker question

Examples

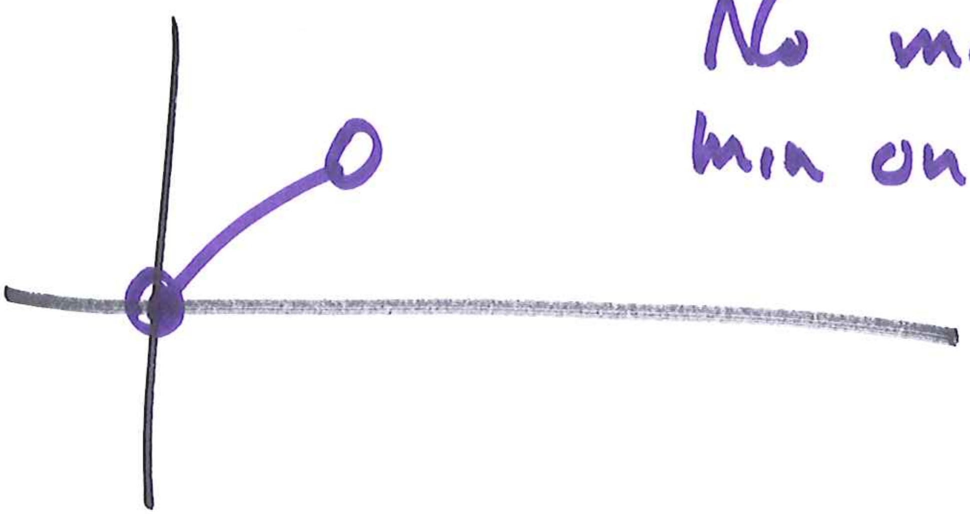
$f(x) = \frac{1}{x}$ on $[1, \infty)$



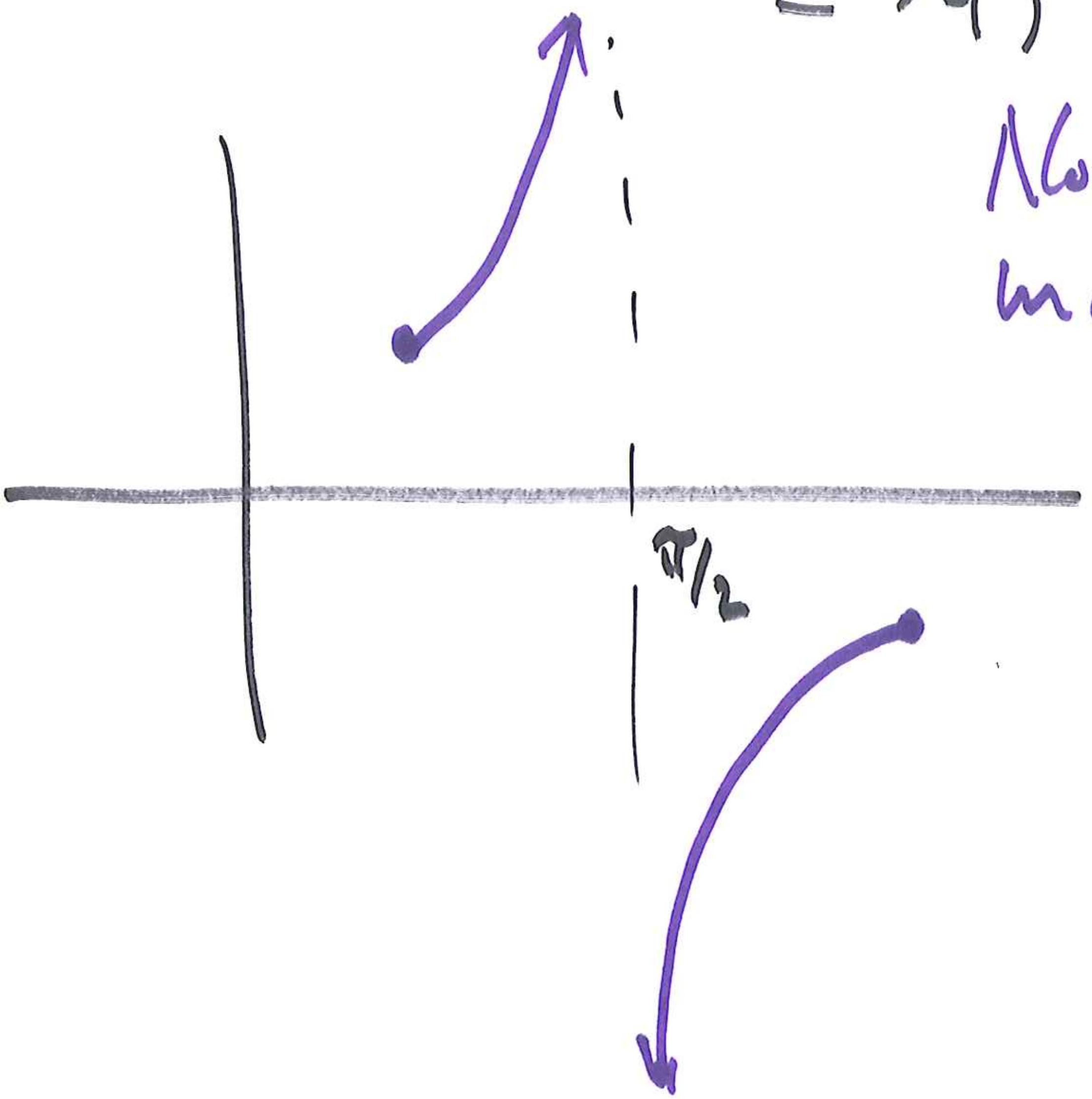
No minimum.

$\sin(x)$ on $(0,1)$

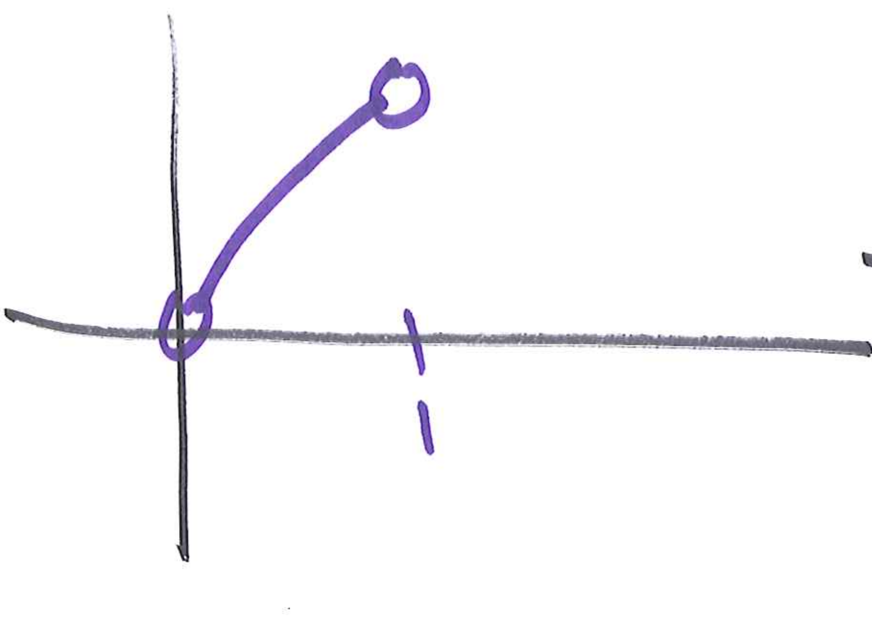
No max or min on $(0,1)$.

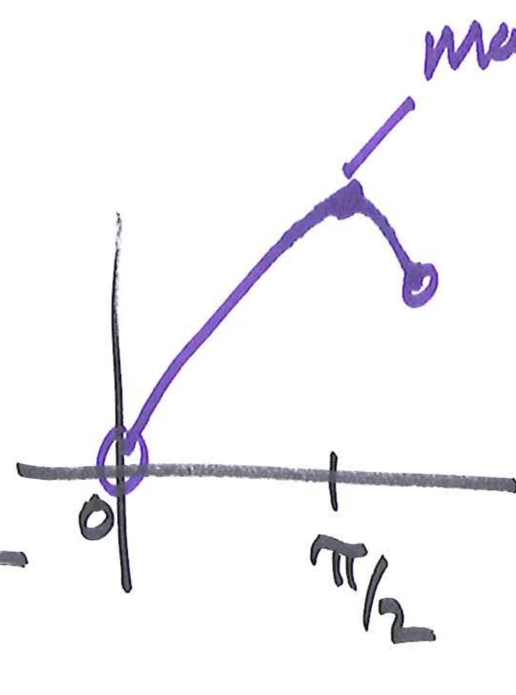


tan(x) on $[\pi/4, 3\pi/4]$



No max or min.





Why?

— It might be on the test.

— Applications.

— Useful in answering the basic question of Calculus — Does the derivative determine a function?

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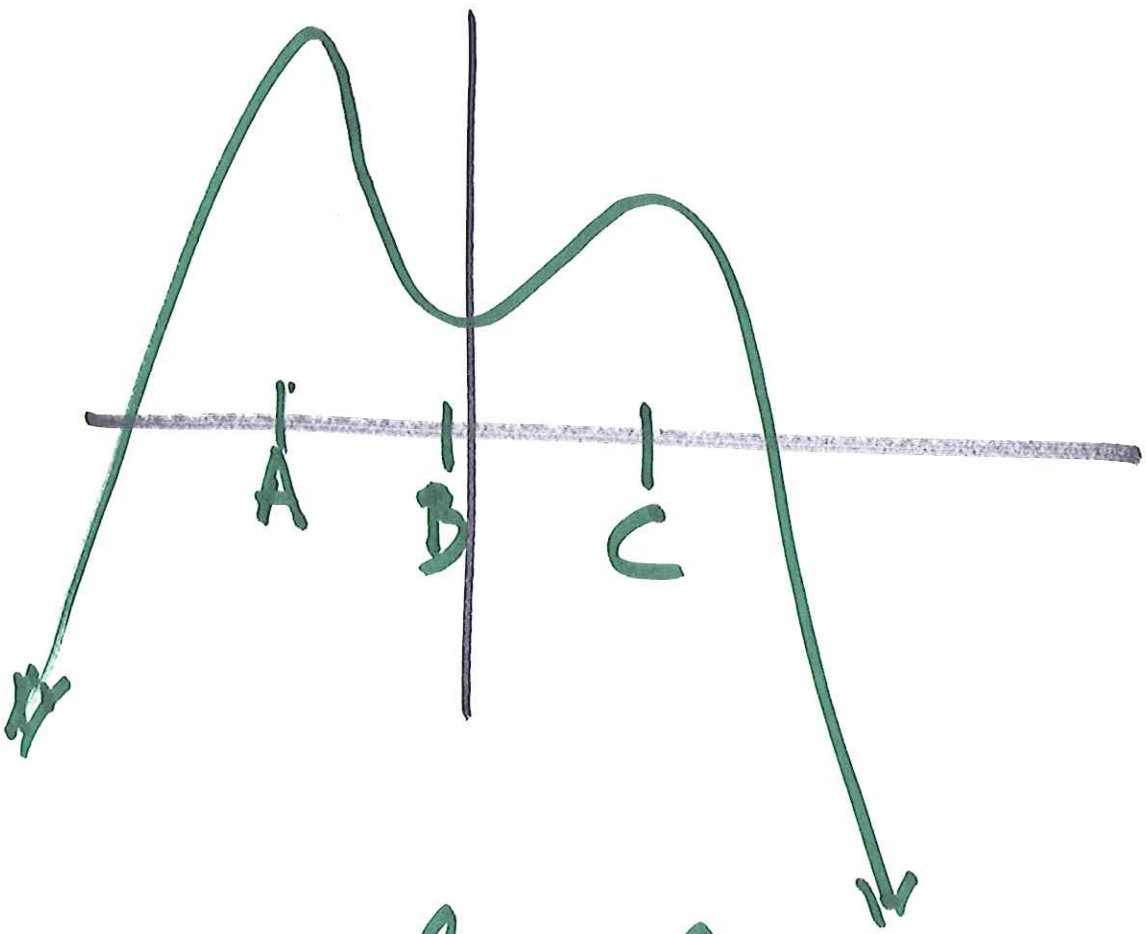
Local maximum. (and minimum.)

Def. If f is a function we say that f has a local maximum at a if there is an open interval I containing a so that $f(a)$ is the absolute maximum on I.

Define local minimum.

Find the absolute extremes and local extremes for the function below.

Extreme = max or min.

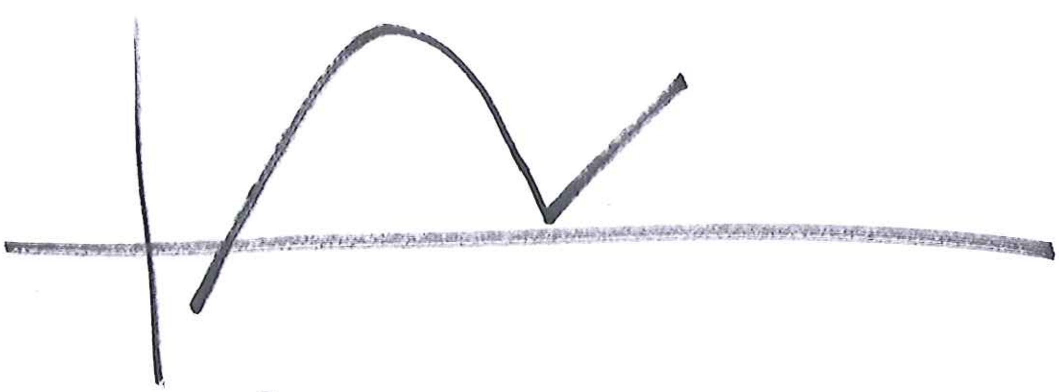


A is a local max & abs. max.

B is a local min.

C is a local max.

<u>Theorem</u>. If f has a local max or min at a, then either $f'(a) = 0$ or $f'(a)$ does not exist.



<u>Why?</u>. ~~For x~~ Suppose $f(a)$ is the max. value for f on I, an open interval.

Then $f(x) \le f(a)$

for all x in I

$$f(x) - f(a) \le 0$$

If $x \le a$, then $x - a \le 0$.

so

$$\frac{f(x)-f(a)}{x-a} \ge 0 \text{ for } x < a.$$

$$\frac{f(x)-f(a)}{x-a} \le 0 \text{ for } x > a.$$

Suppose one sided limits exists

$$\lim_{x \to a^+} \frac{f(x)-f(a)}{x-a} \le 0$$

$$\lim_{x \to a^-} \frac{f(x) - f(a)}{x - a} \geq 0.$$

If $f'(a)$ exists, then $f'(a) \geq 0$ and $f'(a) \leq 0$ so $f'(a) = 0$. $\boxtimes$

<u>Definition</u> We say a is a <u>critical</u> <u>~~number~~ point</u> for f if either $f'(a)=0$ or $f'(a)$ does not exist.

Which of the following is not true? 12

(A) If f is differentiable at 0 and 0 is a local max, then $f'(0)=0$.

(B) If $f'(0)=0$, then 0 is a critical point.

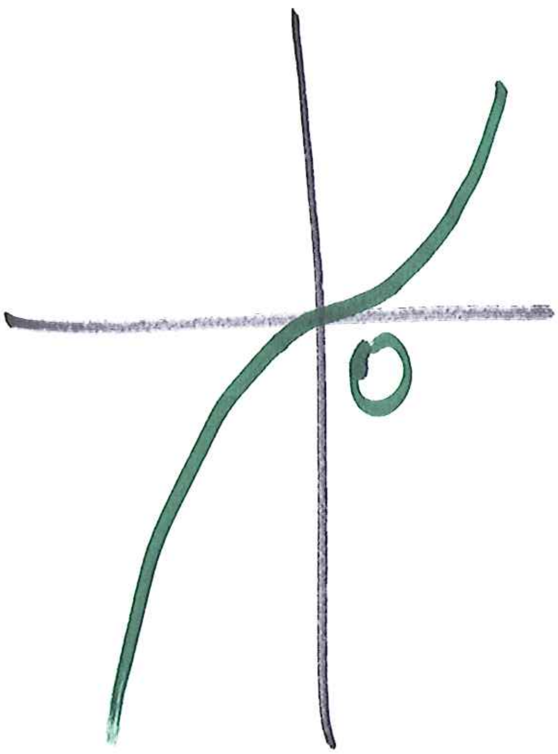
(C) If $f'(0)=0$, then 0 is a local max or min. (False)

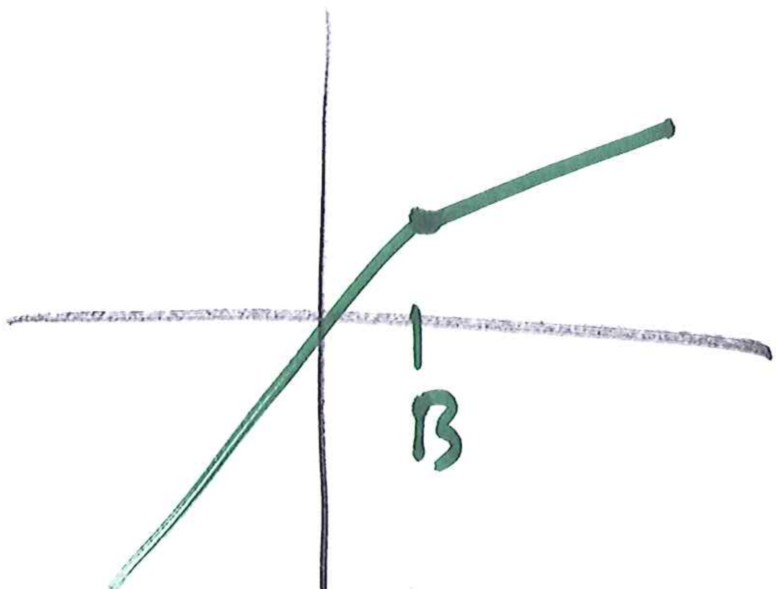
(D) If $f'(0)=0$, then 0 is a critical point.

Example $f(x) = x^3$

$f'(x) = 3x^2$. $f'(0) = 0$.

0 is a critical point but not a max or min





B is a crit. pt. but not a local max or min.

Hunting for absolute extremes.

- If f ~~has a~~ is a continuous on a closed interval.

  - Know there's an abs. max. & abs. min.

  - If the abs. max. is not an endpoint, then it is also a local max. Thus a critical point.

  - Or its an endpoint.

To find absolute extrema

- List endpoints
- Critical points
- Test values
- Choose largest & smallest.

<u>Example</u> Let $f(x) = xe^{-x}$. Find abs. max. & min values on $[0, 2]$.

$$f'(x) = e^{-x} - xe^{-x}$$

$$= (1-x)e^{-x}.$$

$f'(x) = 0$ if $x = 1$.

	List x	$f(x)$	
end	0	0	← smallest
	2	$2/e^2$	
crit	1	$1/e$.	← largest

max. value of $1/e$ at 1.

min. value is 0 at 0.

Let $f(x) = x^3 - 3x$ on $[-3, 1]$. Where is the absolute maximum value of f?

(A) -3

(C) 0

(B) -1

(D) 1