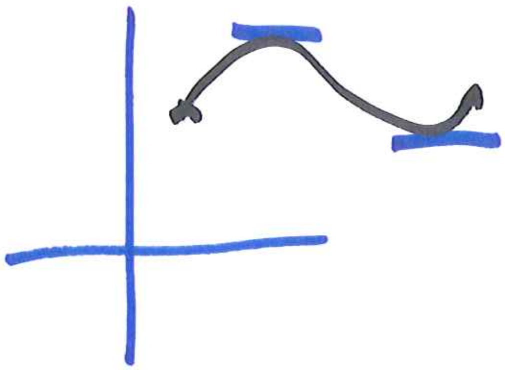
Rolle's Theorem.

If f is continuous on $[a,b]$, differentiable on (a,b), ① then there exists c in (a,b) so that $f'(c) = 0$.

① and $f(a) = f(b)$.

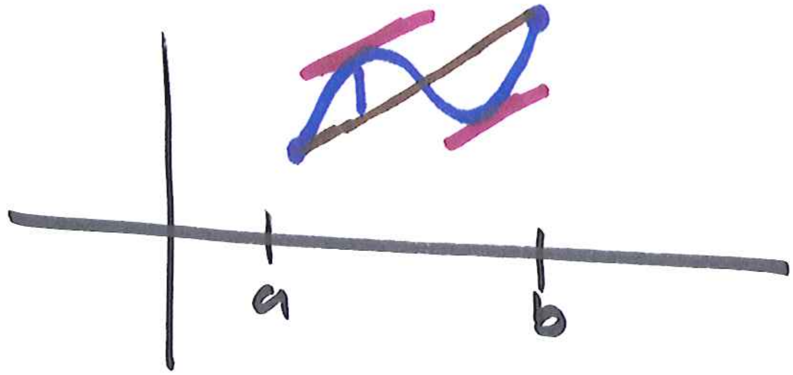
Why? Since f is continuous on $[a,b]$, f has a maximum & minimum. If max & min. both occur at endpoints, then f is constant, so $f'(x) = 0$ everywhere.

If the max. ~~&~~ or min are in (a,b), then f' is 0 at the max. ~~&~~ or min.



Mean-value theorem.

If f is continuous on $[a,b]$ differentiable on (a,b), then there is a value c in (a,b) so that $f'(c) = \frac{f(b)-f(a)}{b-a}$



why?

Take f, subtract a linear fcn. to reduce to Rolle's theorem.

$$L(x) = f(a) + \frac{f(b)-f(a)}{b-a}(x-a).$$

Let $g(x) = f(x) - L(x)$

By Rolle's theorem ($g(a)=g(b)=0$).

$g'(c) = 0$ for some c in (a,b).

$$f'(c) - L'(c) = 0$$

$$f'(c) - \frac{f(b)-f(a)}{b-a} = 0.$$

Corollary If f is differentiable on (a,b) and $f'(x)=0$ for all x in (a,b), then f is constant.

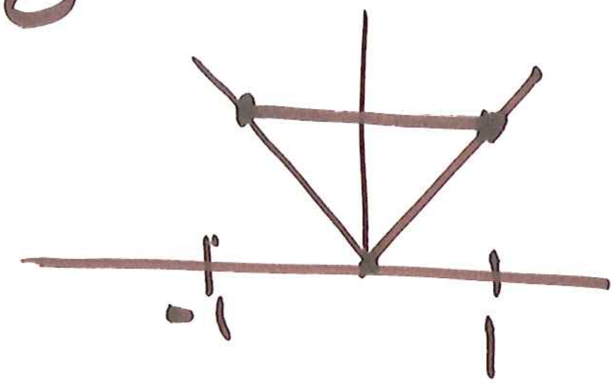
Example. Suppose f is differentiable on the real line. If $f(1)=2$ & $-3 \le f'(x) \le 4$, what is the largest possible value for $f(5)$?

Apply MVT on $[1,5]$

$$-3 \le \frac{f(5)-f(1)}{4} = f'(c) \le 4.$$

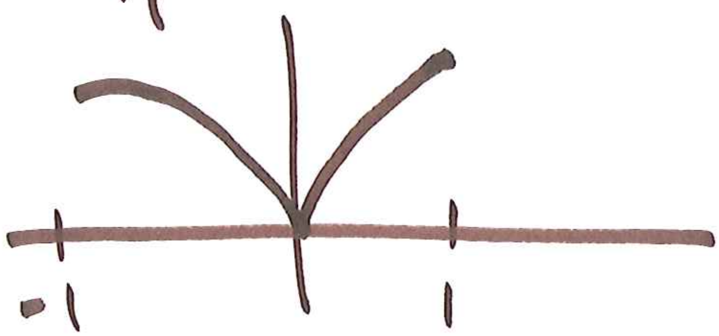
Clicker question

$|x|$ is not differentiable at 0



$\sec(x)$ is not continuous at $\pi/2$.

$|\sin(x)|$



Thus

$$f(5) - f(1) \le 16$$

$$f(5) \le 16 + f(1). = \underline{18.}$$

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A function is increasing on an interval I if whenever x, y are in I with $f(x) < f(y)$, then $f(x) < f(y)$

Theorem. ~~Let~~ f be a differentiable function on an open interval I

- If $f'(x) > 0$ for all x in I, then f is increasing.

• If $f'(x) < 0$ for all x in I, then f is decreasing.

<u>Exercise</u> Define decreasing function.

<u>Why?</u>. Suppose $f'(x) > 0$ on I, a, b are in I with $a < b$. Apply MVT on $[a, b]$

$$\frac{f(b) - f(a)}{(b-a)} = f'(c) > 0$$

$(b-a) > 0$ $\therefore$

$$f(b) - f(a) > 0$$

or $\quad f(b) > f(a)$.

1st derivative test

<u>Theorem</u>. Suppose f is differentiable on (a,b) and c in in (a,b). and

- If $f'(x) < 0$ for $a < x < c$ and $f'(x) > 0$ for $c < x < b$ then c is local min.
- If $f'(x) > 0$ for $a < x < c$ $f'(x) < 0$ for $c < x < b$, then c is a local max.

Example. Let $f(x) = x^3 + 3x^2 - 9x$

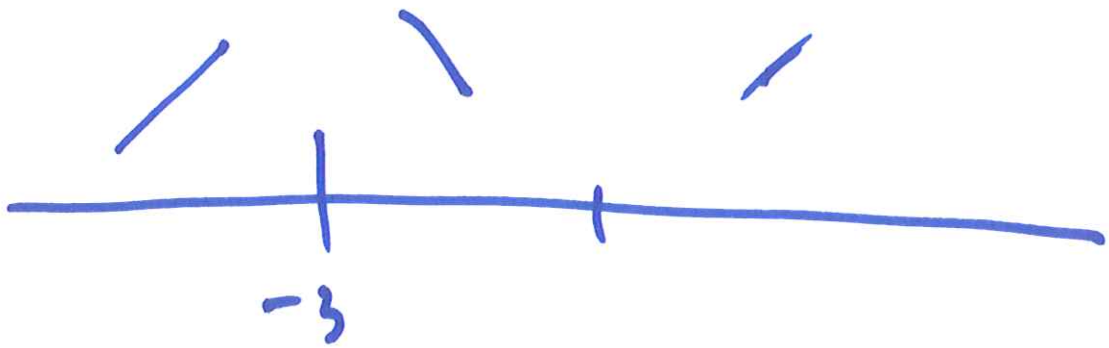
- Find the intervals of increase and decrease.
- Classify critical points as local extrema.

$$f'(x) = 3x^2 + 6x - 9.$$
$$= 3(x^2 + 2x - 3)$$
$$= 3(x-1)(x+3)$$

$f'(x) > 0$ on $(-\infty, -3)$, f increasing

$f'(x) < 0$ on $(-3, 1)$ f decreasing

$f'(x) > 0$ on $(1, \infty)$ f increasing



f has a local max -3

f has a local min at 1.