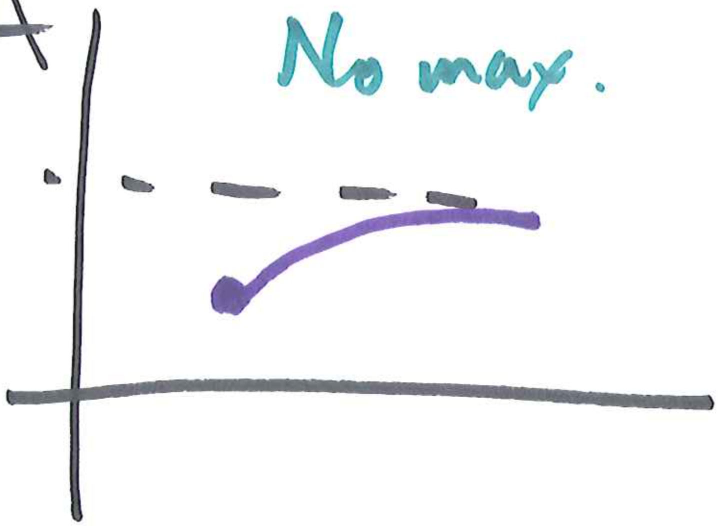
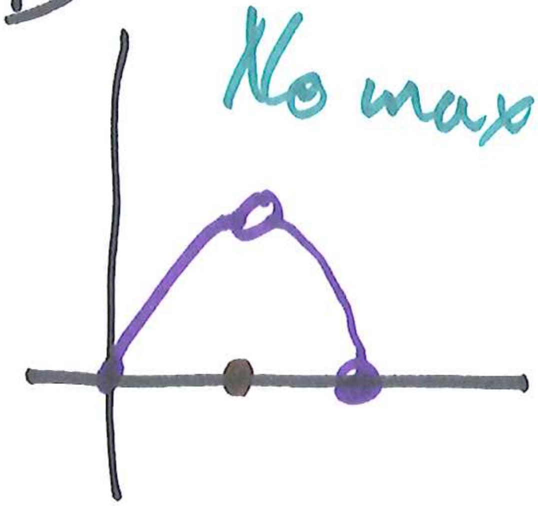
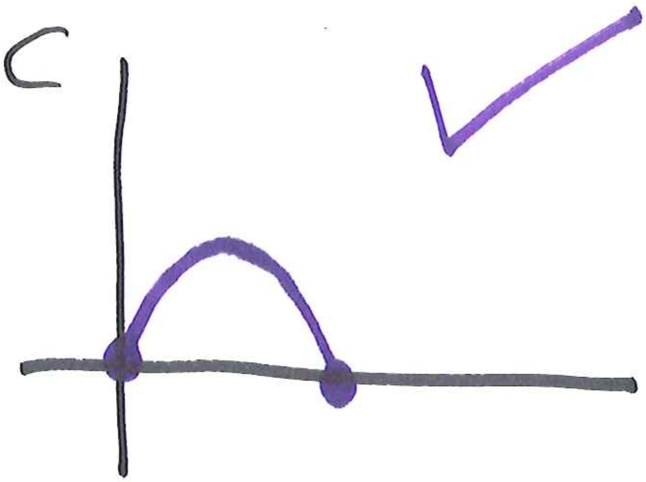
A



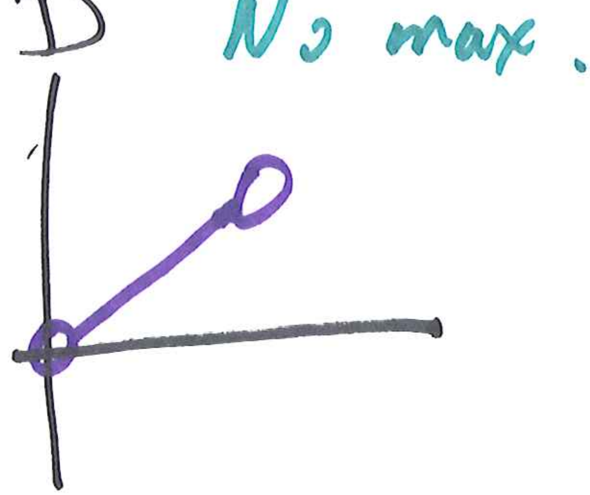
B



C



D



Max-min problems or optimization. L30 11/4/15

— Finding ~~cont~~ max. values on closed + bounded intervals.

– If the function is continuous, the max. exists.

– Max. occurs at a critical point or endpoint.

– List these numbers. Evaluate f to choose largest value.

Finding extreme values on open intervals.
- Special case.

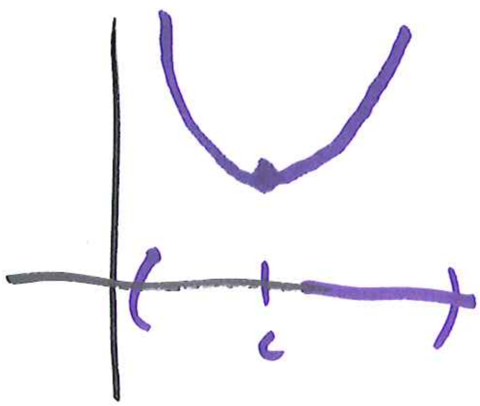
Suppose f is differentiable on an open interval I

If c is in I,

$f'(x) < 0$ for all $x < c$

$f'(x) > 0$ for all $x > c$

Then f has an absolute minimum at c.



1. Read the problem carefully, identify the quantity that we want to make as large or small as possible. This quantity is called the "objective function".
2. Draw a diagram and introduce variables for all quantities from the problem.
3. Write an expression for the objective function. This expression may involve more than one variable.
4. Write the constraint equations. The constraint equation is an equation relating the quantities in the problem. Use the *constraint equation* to eliminate extra variables from our objective function.

5. Write clearly the function (of one variable) to be optimized and state the domain. The domain may be smaller than the natural domain where the function is defined. These restrictions on the domain may be stated in the problem or may arise because of geometric reality that a negative length is not likely to occur.

6. Find the extreme value of the objective function using one of the tests above. Explain why you know you have found the maximum.

7. Answer the question. Are you to give the location of the maximum, the extreme value, or do you need to compute additional quantities?

Optimization problem —
Find the smallest of

$f(x) = x + \frac{5}{x}$ for $x > 0$
or $(0, \infty)$

Compute $f'(x) = 1 - \frac{5}{x^2}$

$f'(x) > 0$ if $\frac{5}{x^2} < 1$

< 0 if $\frac{5}{x^2} > 1.$

Solving $\frac{5}{x^2} < 1$, gives
$x > \sqrt{5}$ or ~~$x < -\sqrt{5}$~~

Solving $\frac{5}{x^2} > 1$ gives $x < \sqrt{5}$.

Example Suppose the product of two positive numbers is 5. Find the largest & smallest values for the sum.

Two numbers x, y

~~Constraint~~ Objective function is the sum.

$$x+y.$$

Constraint $- \quad xy = 5$ or

$y = 5/x.$

Eliminate y.

Objective becomes $x + \frac{5}{x}$

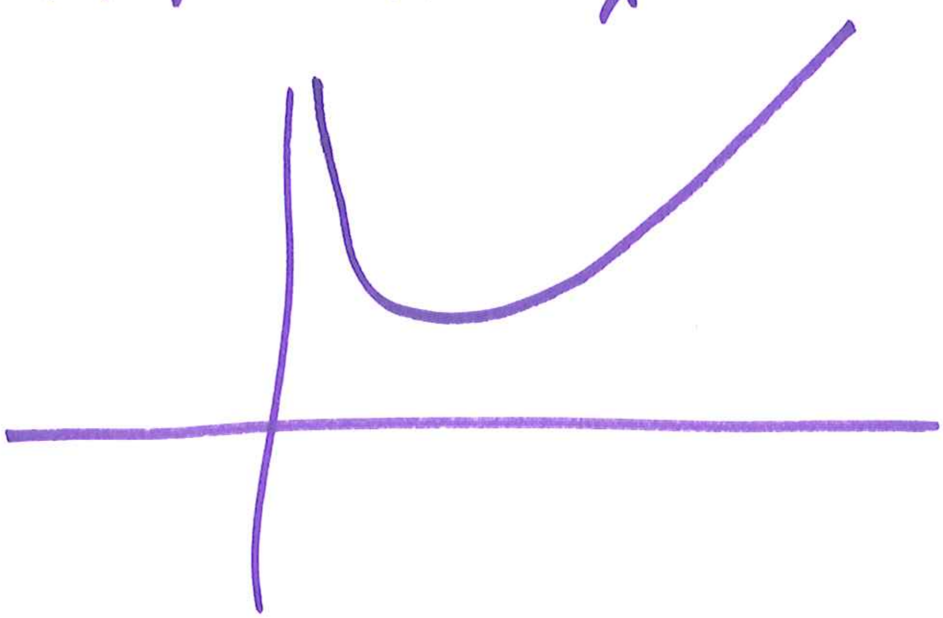
Thus f has an ~~min~~ absolute min. at $x=\sqrt{5}$.

Smallest value for the sum is $f(\sqrt{5}) = 2\sqrt{5}$.

Largest value?

Finding max. value of

$f(x) = x + \frac{5}{x}$ on $(0,\infty)$

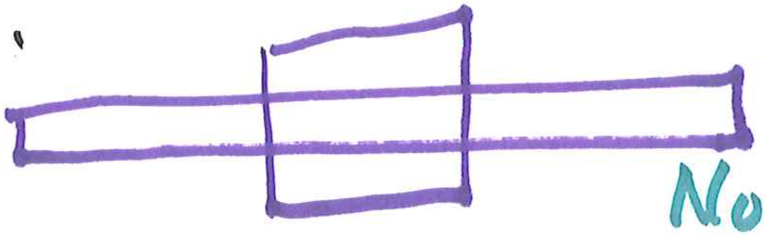


No max.

Find the largest value of xy if $x+y=5$.
$x \geq 0$ & $y \geq 0$

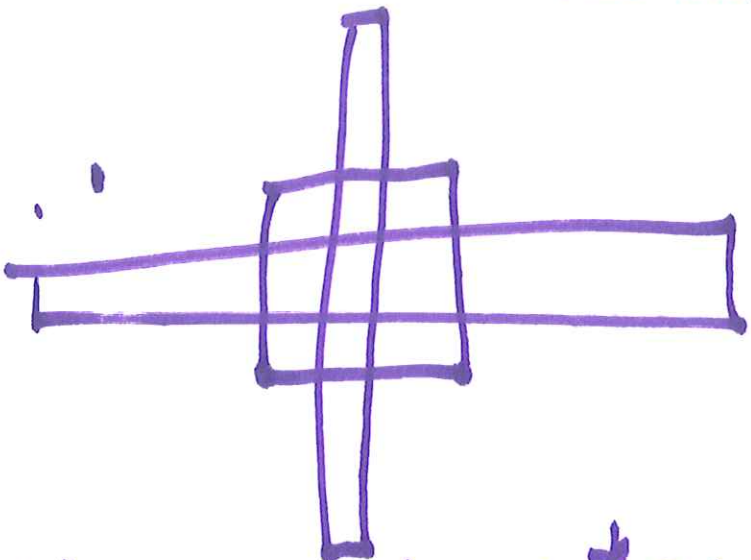
~~FDA~~ ~~xy~~ $x+y=5$.
$y = 5-x$. max. value
of $xy = x(5-x)$,
x is in $[0,5]$.

A.



No max perimeter

B.

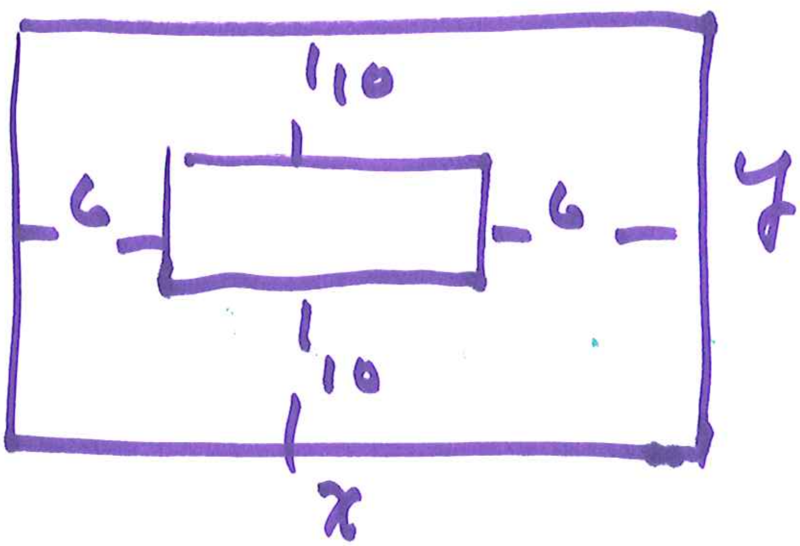


- min. perimeter at a square.

C. - Max. area a square

D. 4

Example. A rectangular poster has area of 6000 cm^2. The poster is to consist of a rectangular printed area in the center with 10 cm margins top and bottom and 6 cm margins left and right. Find the dimensions of the poster with the largest printed area.



$xy = 6000$

~~Area is~~ printed area is

$(x-12)(y-20)$

Set $y = \frac{6000}{x}$.

Goal is to max.

$$f(x) = (x-12)\left(\frac{6000}{x} - 20\right)$$

On domain. for $x \geq 12$

Also we need $y \geq 20$

$\frac{6000}{x} \geq 20$, $x \leq 300$.

Domain is $[12, 300]$.

$$f(x) = 6000 - \frac{72{,}000}{x} + 240$$
$$-20x$$

$$f'(x) = \frac{72{,}000}{x^2} - 20.$$

$f'(x) = 0$ at 60.

Crit. # at 60

Endpoints of 12 to 300

x	$f(x)$	
12	0	
60	3840	$\leftarrow$ max.
300	0	