L34/11/13

Consider the region $R = \{(x,y) : 0 \le y \le f(x),\ 1 \le x \le 2\}$ with $f(x) = 3 + 5x$.

- Find the right sum R_n which approximate the area of R.
- Find the area as $\lim_{n \to \infty} R_n$.
- Check your answer using the formula for the area of a trapezoid.

Area of trapezoid R. is $\frac{1}{2}(8+18)\cdot 2$

$= \underline{26}$

$R = \{(x,y) : 0 \le y \le 3+5x,\ 1 \le x \le 2\}$

Divide $[1,3]$ into n equal intervals of length

$\Delta x = \frac{3-1}{n} = 2/n$. The division points will be $x_k = 1 + k\Delta x$

$= 1 + 2k/n$. Area of k^{th} rectangle (orange rectangle in pictures)

$$\Delta x f(x_k) = \frac{2}{n}(3+5x_k).$$

$$= \frac{2}{n}\left(3+5\left(1+\frac{2k}{n}\right)\right)$$

$$= \frac{2}{n}(8+10k/n).$$

Thus $R_n = \sum_{k=1}^{n} \frac{2}{n}\left(8+\frac{10k}{n}\right)$.

$$R_n = \sum_{k=1}^{n} \frac{2}{n}\left(8 + \frac{10k}{n}\right).$$

$$= \sum_{k=1}^{n}\left(\frac{16}{n} + \frac{20k}{n^2}\right)$$

$$= \frac{16}{n}\sum_{k=1}^{n} 1 + \frac{20}{n^2}\sum_{k=1}^{n} k.$$

$$= \frac{16}{\cancel{n}}\cancel{n} + \frac{20}{n^{\cancel{2}}}\cdot\frac{\cancel{n}(n+1)}{2}$$

Known $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

$$\lim_{n\to\infty}\left(16 + 10\left(\frac{n+1}{n}\right)\right) = 16 + 10$$

$$= \underline{26}.$$

This agrees with our answer using the area of a trapezoid.

C4.1 #10

Estimate the value using the Linear Approximation to e^x at $x = 0$ and then find the error using a calculator.

$$e^{+0.17}$$

The error is

The linear approximation at 0 is $L(x) = f(0) + f'(0)(x-0)$

$= 1 + x$

Since $f(0) = f'(0) = e^0 = 1$.

We use $e^{0.17} \approx L(0.17) = 1.17$.

The error is $|e^{0.17} - L(0.17)| \approx 0.01535$

C 4.2 #7

Find all critical points of the function $f(x) = x^3 - 3x^2 - 3x$ in the interval $(-2, 1)$.

The critical point(s) is/are

Find the absolute maximum and minimum values of the function f on the interval $[-2, 1]$.

The absolute minimum is

The absolute maximum is

- The critical points are points where $f'(x) = 0$ or $f'(x)$ is undefined.
- We compute

$$f'(x) = 3x^2 - 6x - 3$$
$$= 3(x^2 - 2x - 1)$$

$f'(x) = 0$ if

$$x = \frac{2 \pm \sqrt{4+4}}{2}$$
$$= 1 \pm \frac{1}{2}\sqrt{4 \cdot 2}$$
$$= 1 \pm \frac{1}{2}\sqrt{4}\sqrt{2} = 1 \pm \sqrt{2}$$

$1+\sqrt{2}$ is not in the interval $(-2, 1)$.

On a closed interval, we know there is a largest and smallest value and it occurs at a critical point or an endpoint. Thus we need to compute f at $-2, 1-\sqrt{2}, 1$

x	$f(x)$
-2	-14
$1-\sqrt{2}$	$0.65685\ldots$
1	-5

The largest value is approximately 0.66 at $x = 1-\sqrt{2}$. The absolute min. value is -14 at $x=-2$.

C4.3 #1

Find a point c satisfying the conclusion of the Mean Value Theorem for the following function and interval.

$f(x) = x^{-5}, \qquad [1,3]$

c =

According to the mean value theorem there is a point c in (1, 3) so that

$$\frac{f(3)-f(1)}{3-1} = f'(c)$$

Thus we want

$$\frac{\frac{1}{3^5}-1}{3-1} = -5 \cdot \frac{1}{c^6}$$

or

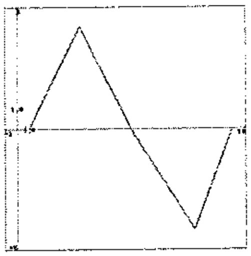
$$\frac{\frac{1}{243}-1}{2} = -\frac{121}{243} = -5/c^6 \text{ or}$$

$$c^6 = 1215/121, \quad c = \sqrt[6]{1215/121}$$

$$\approx 1.4688$$

C4.4 #6

The figure below is the graph of the derivative f' of a function f.



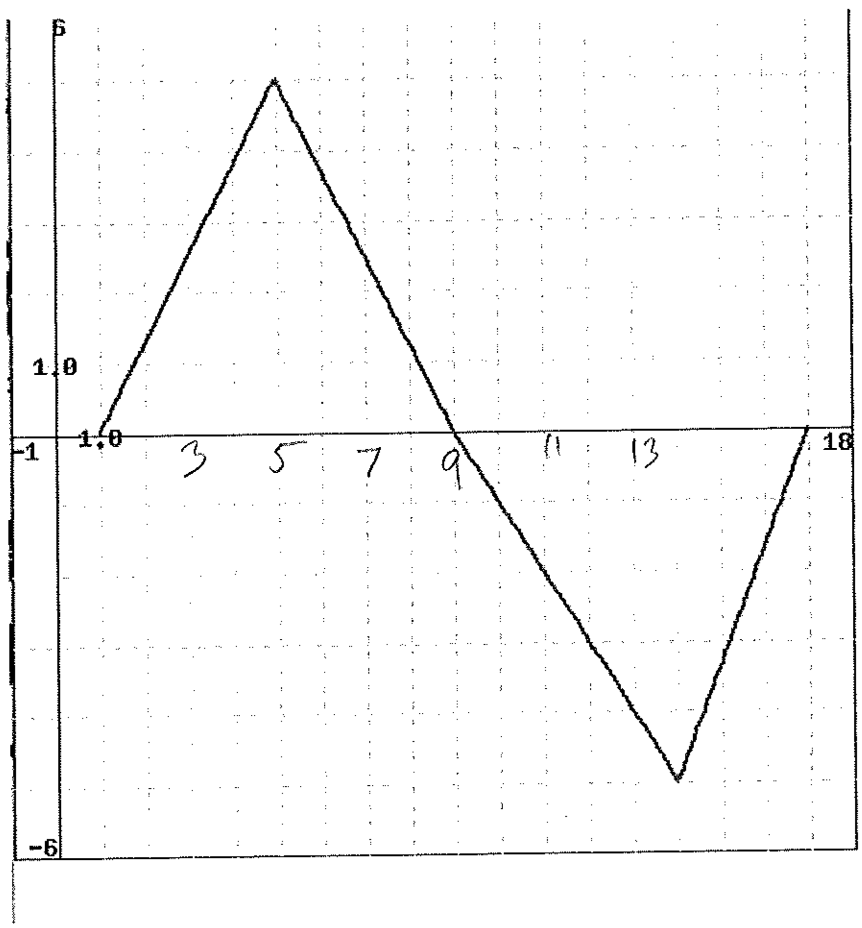
Give the x-coordinate(s) of the inflection point(s) of f.

help (numbers)

If there is more than one inflection point, enter them as a comma separated list.

On which interval(s) is f concave down?

help (intervals)



C 4.3 #6

A function is concave up if f' is increasing and concave down if f' is decreasing.

From the graph we say

f is concave up on $(1, 5)$

f is concave down on $(5, 14)$

f is concave up on $(14, 17)$

f has inflection points at $x = 5$ at $x = 14$.

2.7 #6

Find the horizontal asymptotes of $f(x) = \frac{\sqrt{6x^2+7}}{8x+6}$.

(Use symbolic notation and fractions where needed. Give your answer as comma separated list.)

The horizontal asymptotes for $f(x)$ are

$y =$ $\sqrt{6}/8, -\sqrt{6}/8$ help (fractions)

The line $y=c$ is a horizontal asymptote for f if

$\lim_{x\to\infty} f(x) = c$ or $\lim_{x\to-\infty} f(x) = c$.

That we compute limits at ∞

$$\frac{\sqrt{6x^2+7}}{8x+6} = \frac{|x|\sqrt{6+7/x^2}}{x\,(8+6/x)}.$$

$$\lim_{x\to\infty} \frac{|x|}{x}\,\frac{\sqrt{6+7/x^2}}{8+6/x} = 1\cdot\sqrt{6}/8$$

$$\lim_{x\to-\infty} \frac{|x|}{x}\,\frac{\sqrt{6+7/x^2}}{8+6/x} = -1\sqrt{6}/8$$

4.5 #7

Apply L'Hôpital's Rule to evaluate the following limit. It may be necessary to apply it more than once.

$$\lim_{x \to 1} \frac{x-1}{e^x - e} =$$

At $x=1$, the expression $\frac{x-1}{e^x-e}$ gives the indeterminate form $0/0$.

Thus we may use L'Hôpital to find

$$\lim_{x \to 1} \frac{x-1}{e^x-e} = \lim_{x \to 1} \frac{1}{e^x} = \frac{1}{e}.$$