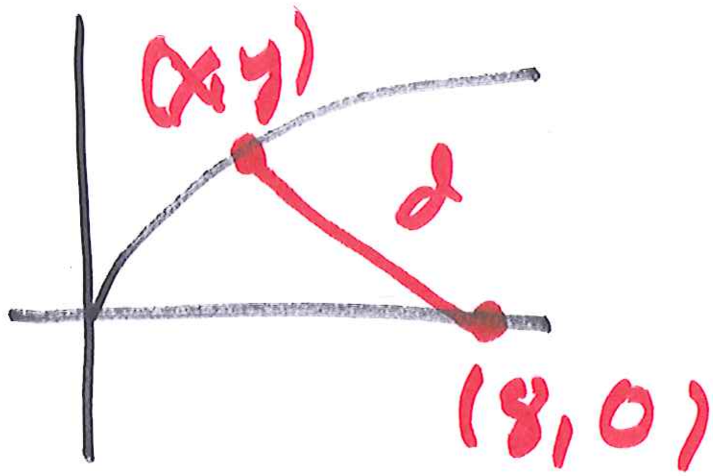
L35/Review.

4.7 #5

Find the point P on the graph of the function $y = \sqrt{x}$ closest to the point $(8, 0)$

The x coordinate of P is: .



minimize $d = \sqrt{(x-8)^2 + (y-0)^2}$

Use $y = \sqrt{x}$.

$= \sqrt{(x-8)^2 + (\sqrt{x})^2}$

Easier to minimize d^2 than d.

~~Minimize~~

$f(x) = d^2 = (x-8)^2 + x.$

for $x \geq 0$

~~Compute $f'(x) =$~~

Simplify $x^2 - 16x + 64 + x.$

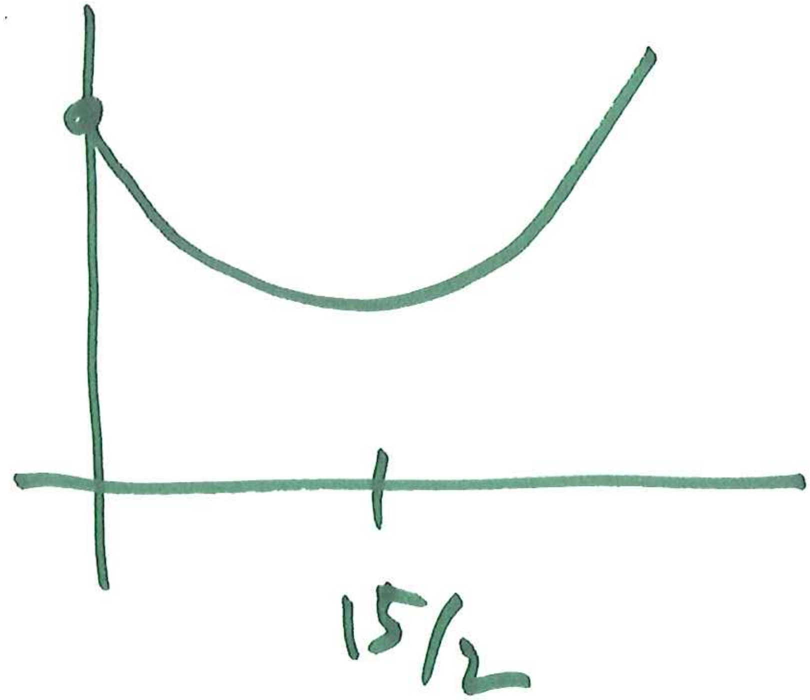
$= x^2 - 15x + 64.$

$f'(x) = 2x - 15.$

$f'(x) > 0$ if $x > \cancel{7/2}$ $15/2$

$= 0$ if $x = \cancel{7/2}$ $15/2$

< 0 if $x < \cancel{7/2}$. $15/2.$



Min. occurs at

$P = \left(15/2, \sqrt{15/2}\right)$.

- Justify.

Since f is increasing for $x > 15/2$ & decreasing for all $x < 15/2$, $x = 15/2$ must give a minimum.

4.8 #2

Apply Newton's method to $p(x) = x^2 - 7x - 10$ with initial guess $x_0 = 11$ to calculate three successive improved guesses for a solution of $p(x) = 0$.

$x_1 =$

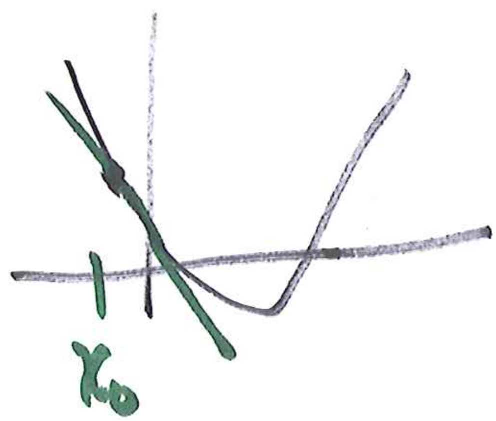
$x_2 =$

$x_3 =$

Recall the iteration for Newton's method -

$$x_{n+1} = x_n - f(x_n)/f'(x_n).$$

$$p'(x) = 2x - 7.$$



$$x_1 = x_0 - \frac{p(x_0)}{p'(x_0)}$$

$$= 11 - \frac{121-77-10}{15}$$

$$\approx 8.733$$

Again.

Work needed on exam.

1. Iteration formula.

2. Iterates.

4.9 #9

Evaluate this indefinite integral.

$$\int \frac{3}{5}\sin(x) - \frac{1}{4}\cos(x)\,dx =$$

Know $\int \sin(x)\,dx = -\cos(x) + C$

$\int \cos(x)\,dx = \sin(x) + C_1$

$$\int \frac{3}{5}\sin(x) - \frac{1}{4}\cos(x)\,dx$$

$$= \frac{3}{5}\int \sin(x)\,dx - \frac{1}{4}\int \cos(x)\,dx$$

$$= -\frac{3}{5}\cos(x) - \frac{1}{4}\sin(x) + C.$$

5.1 #9

Use linearity to evaluate the sum.

$$\sum_{m=1}^{20}\left(35+\frac{45m}{2}\right)^2 =$$

$$\sum_{m=1}^{20}\left(35+\frac{45m}{2}\right)^2$$

$$= \sum_{m=1}^{20}\left(35^2 + 35\cdot 45m + \left(\frac{45}{2}\right)^2 m^2\right)$$

$$= 35^2\sum_{m=1}^{20} 1 + 35\cdot 45\sum_{m=1}^{20} m \cancel{+ 45 \sum} + \left(\frac{45}{2}\right)^2\sum_{m=1}^{20} m^2$$

$$= 35^2 \cancel{35^2}$$

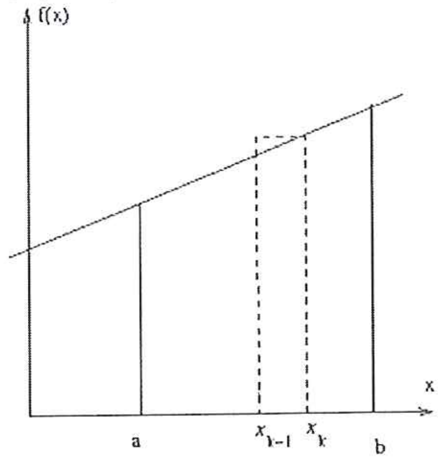
$$= 35^2 \cdot 20 + 35 \cdot 45 \frac{20 \cdot (21)}{2}$$

$$+ \left(\frac{45}{2}\right)^2 \frac{(20)(21)\cdot(41)}{6}$$

Use

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

5.1 #12



We want to find the area of a region S which lies below the graph of $f(x) = 8x + 9$ and above the interval $[a, b] = [4, 6]$ on the x-axis. Thus $S = \{(x, y) : 4 \le x \le 6, 0 \le y \le 8x + 9\}$.

To do this we begin by dividing the interval $[4, 6]$ into N equal subintervals using the points $x_0 = 4, x_1, x_2, \ldots, x_N = 6$.

The length of each sub-interval $[x_{k-1}, x_k]$ is $2/N$.

Find a formula for the kth division point in terms of k and N,

$x_k =$. help (formulas)

$4 + \frac{k2}{N}$

Let A_k be the rectangle w/ base $[x_{k-1}, x_k]$ and height $f(x_k)$ (this is a right sum)

Area of A_k is

$$\Delta x \, f(x_k).$$

$$= \frac{2}{N}\left(8\left(4+\frac{2k}{N}\right)+9\right).$$

$$= \frac{2}{N}\left(41+\frac{16k}{N}\right)$$

$$= \frac{82}{N} + \frac{32k}{N^2}.$$

Sum areas of all A_k's $k=1$ N.

$$\sum_{k=1}^{N}\left(\frac{82}{N}+\frac{32k}{N^2}\right)$$

Simplify:

$$\frac{82}{N}\sum_{k=1}^{N} 1 + \frac{32}{N^2}\sum_{k=1}^{N} k.$$

Given $\sum_{k=1}^{N} k = \frac{N(N+1)}{2}$.

$$= \frac{82}{N} \cdot N \cdot + \frac{32}{N^2} \frac{N(N+1)}{2}$$

$$= 82 + \frac{32}{2} \frac{(N+1)}{N}.$$

Take limit to find area —

$$\lim_{N \to \infty} \left(82 + 16 \frac{N+1}{N} \right).$$

$$= 82 + 16 \cdot 1 = 98.$$

Clicker #3.

$$\sum_{k=1}^{49}(2k+1) = 2\sum_{k=1}^{49} k + \sum_{k=1}^{49} 1$$

$$= 2\,\frac{(49)(50)}{2} + 49.$$

$$= 49\cdot 50 + 49$$

$$= 2450 + 49$$

$$= 2499.$$

5.1 #12 pt2

Find an expression of the sum of the areas A_k,

$$S_N = \sum_{k=1}^{N} A_k.$$

$S_N =$

.

It will be helpful to use the formulas for the sums of powers of integers from the textbook.

Take the limit as N tends to infinity to find the area of S.

$S = \lim_{N\to\infty} S_N =$.