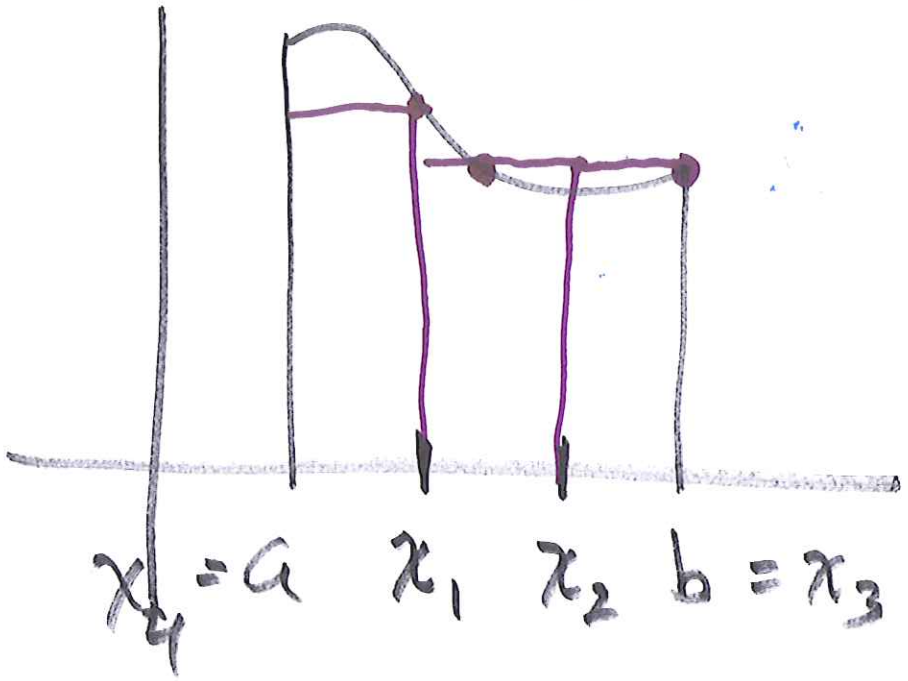
L36

- Questions about grading exams? Write question on front of exam & return to TA.

- Course full? Send me an email & wait a day.

- Section full? Register for another section & check again later.

- This sum represents the area if $f \geq 0$ & $a \leq b$.



Example Let $f(x) = 1/x$ on $[1, 3]$, $P = \{1, 5/4, 3/2, 2, 3\}$ $C = \{1, 3/2, 7/4, 5/2\}$. Find $R(f, P, C)$.

- Notation $\Delta x_i = x_i - x_{i-1}$

Interval	Δx_i	$f(t_i)$	Area
$[1, 5/4]$	$1/4$	1	$1/4$
$[5/4, 3/2]$	$1/4$	$2/3$	$1/6$
$[3/2, 2]$	$1/2$	$4/7$	$2/7$
$[2, 3]$	1	$2/5$	$2/5$

$1.1024\ldots$

Clicker $f(x) = 2x$

Interval	Δx_i	t_i	$f(t_i)$	Area
$[1, 3/2]$	$1/2$	1	2	1
$[3/2, 2]$	$1/2$	2	4	2

3

The integral.

Riemann sums.

- Given a function f on an interval $[a, b]$.
- $P = \{x_0 < x_1 <$
  $= \{a = x_0 < x_1 < x_2 \dots < x_n = b\}$.
- P is called a partition.
- $C = \{t_1 \dots t_n\}$ with $t_i \in [x_{i-1}, x_i]$ are called sample points.
- The Riemann sum $R(f, P, C)$ is

$$\sum_{i=1}^{n} f(t_i)(x_i - x_{i-1}).$$

Define the integral

~~Mesh of a partition~~

~~Δ ||P||~~

Norm of a partition.

$||P|| = \max(\Delta x_1, \ldots, \Delta x_n)$

is the width of largest subinterval.

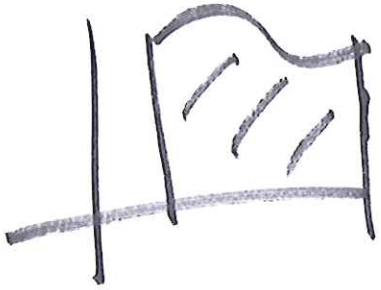
$\int_a^b f(x)dx$ is

$\lim_{||P||\to 0} R(f, P, c) = \int_a^b f(x)dx.$

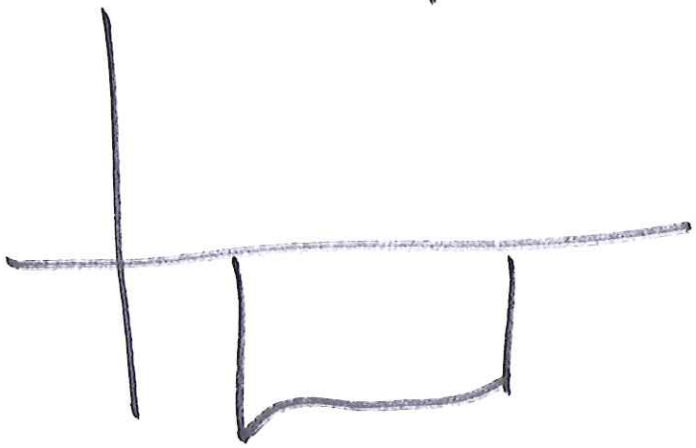
This called the definite integral of f on $[a,b]$.

Theorem If f is continuous on $[a,b]$, then $\int_a^b f(x)\,dx$ exists.

- Geometric interpretation.
  - If $f \geq 0$, $a \leq b$, $\int_a^b f\,dx$ represents the area enclosed by $y = f(x)$, $y = 0$, $x = a$, $x = b$.

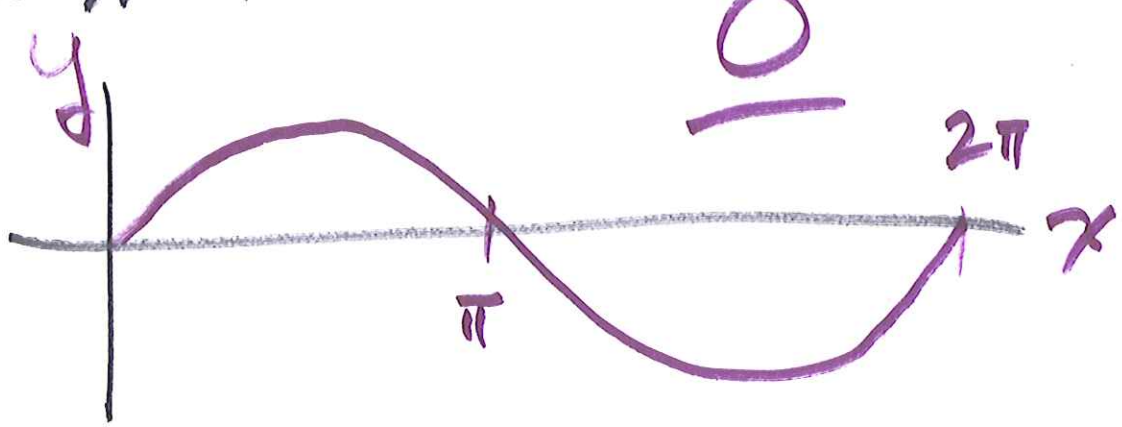


If $f \le 0$, $a \le b$ then $\int_a^b f(x)\,dx$ is the negative of the area



Example. $\int_0^{2\pi} \sin(x)\,dx$

Guess the value.



Properties

$$\int_a^b c\,dx = c(b-a).$$

Since $\sum_{i=1}^{n} c(x_i - x_{i-1})$

$$= c(x_n - x_0)$$

$$= c(b-a)$$

$$\int_a^b f(x) \pm g(x)\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$$

Since $\sum (f(t_i) \pm g(t_i))\Delta x_i$

$$= \sum f(t_i)\Delta x_i \pm \sum g(t_i)\Delta x_i$$

$$\int_a^b c f(x)\,dx = c\int_a^b f(x)\,dx.$$

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx.$$

Example. Find

$\int_2^3 (2+3x)\,dx$ if given that

$$\int_0^a x\,dx = a^2/2$$

$$\int_2^3 2+3x\,dx = \int_2^3 2\,dx + 3\int_2^3 x\,dx$$

$$= 2 + 3\int_2^3 x\,dx$$

Oops - One more property -

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^1 f(x)\,dx.$$

$$\int_0^3 x\,dx = \int_0^2 x\,dx + \int_2^3 x\,dx.$$

$$9/2 = 2 + \int_2^3 x\,dx.$$

$$\int_2^3 x\,dx = \underline{5/2}.$$

$$= 2 + 3\int_2^3 x\,dx$$

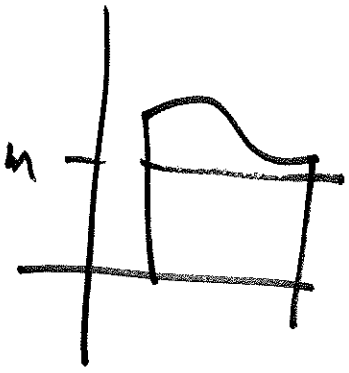
$$= 2 + 3 \cdot 5/2 = 2 + 15/2.$$

Comparison -

If $a \le b$, and $f(x) \ge m$

then

$$\int_a^b f(x)\,dx \ge m(b-a)$$



If $a \le b$, $f(x) \le M$

$$\int_a^b f(x)\,dx \le M(b-a).$$

Clicker

$\int_1^2 f(x)\,dx = 2, \int_1^3 f(x)\,dx = 5.$

Find $\int_3^2 f(x)\,dx.$

$$\int_1^2 f(x)\,dx + \int_2^3 f(x)\,dx = \int_1^3 f(x)\,dx$$

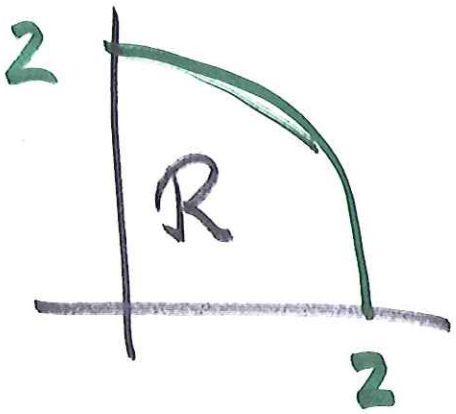
$$2 + \int_2^3 f(x)\,dx = 5$$

$$\int_2^3 f(x)\,dx = 3$$

$$\int_3^2 f(x)\,dx = -\int_2^3 f(x)\,dx = -3$$

Geometry → Integrals

$$\int_0^2 \sqrt{4-y^2}\, dx = ?$$



R is a
1/4 - circle.
of radius 2.

Full circle has area
4π

1/4 - circle has area π.