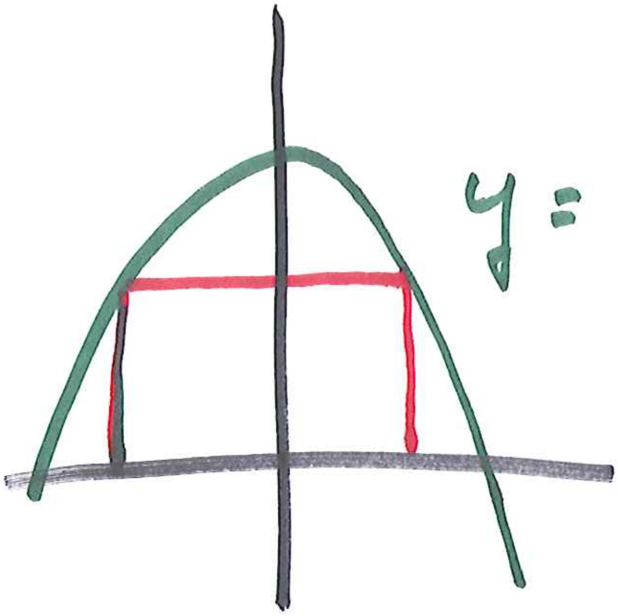
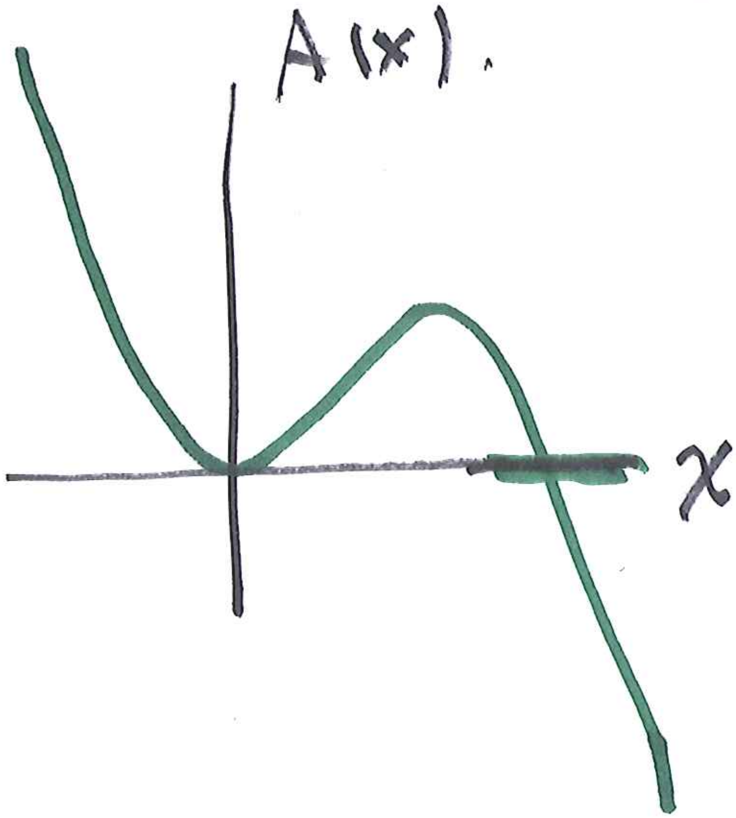


$y = 15 - x^2$

$$A(x) = 2x(15 - x^2)$$
$$= 30x - 2x^3$$



want max value on $[0, \sqrt{15}]$

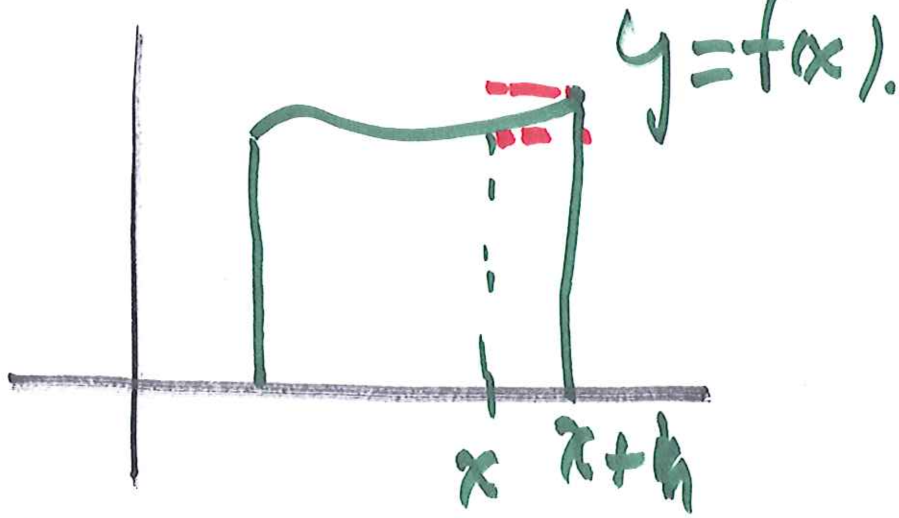
Example.

$$F(x) = \int_0^x \sin(t^3)\,dt$$

$\sin(t^3)$ is continuous for t in $(-\infty, \infty)$ and thus FTC II tells us

$$F'(x) = \sin(x^3).$$

why?



$F(x) = \int_a^x f(t)\,dt.$ To find F', consider

$$\frac{1}{h}\Big(F(x+h) - F(x)\Big).$$

$$= \frac{1}{h}\int_x^{x+h} f(t)\,dt.$$

— Assume $h > 0$. f is continuous on $[x, x+h]$.

Let M_h = max of f on $[x, x+h]$

m_h = min of f on $[x, x+h]$.

Since f is continuous

$$\lim_{h \to 0^+} m_h = \lim_{h \to 0^+} M_h = f(x).$$

So consider

$$\frac{1}{\cancel{h}} \cancel{h} m_h \le \frac{1}{h} \int_x^{x+h} f(t)\,dt. \le \frac{1}{\cancel{h}} \cancel{h} M_h$$

Take limit as $h \to 0$ to squeeze theorem ~~the~~ tells

$$\underbrace{\lim_{h \to 0^+} \frac{1}{h} \int_x^{x+h} f(t)\,dt}_{= F'(x)} = f(x)$$

Example Find

$$\frac{d}{dx}\int_0^{x^2}\sqrt{1+t^2}\,dt.$$

- Use chain rule
- FTC II.

Set $F(s)=\int_0^s \sqrt{1+t^2}\,dt.$

Find $\frac{d}{dx}F(x^2).$

$$= 2x\,F'(x^2) = 2x\sqrt{1+x^4}.$$

$$F'(s) = \sqrt{1+s^2}$$

Fundamental Theorem Calculus part ii.

<u>Theorem</u>. If f is a continuous function on an open interval I, and a is in I, then

$$F(x) = \int_a^x f(t)\,dt$$

is differentiable for x in I.

$$F'(x) = f(x).$$

Example.

$$\frac{d}{dx}\int_x^1 e^{t^2}\,dt.$$

Prop. of integral.

$$= \frac{d}{dx} - \int_{\cancel{x}\,1}^{x} e^{t^2}\,dt$$

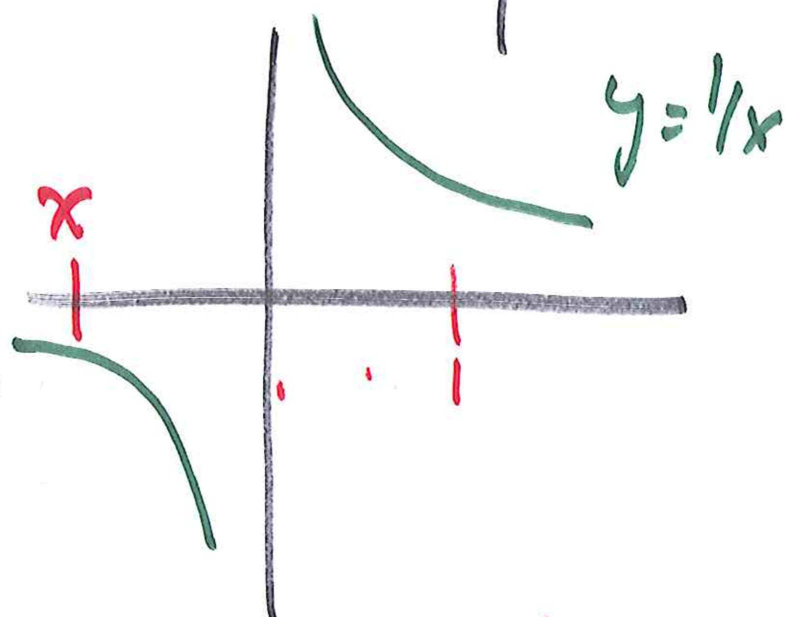
$$= -e^{x^2}.$$

Use FTC II

Example Let

$$L(x) = \int_1^x \frac{1}{t}\,dt.$$

- where is L increasing?
- What is the domain of L?



L is defined if $1/t$ is continuous on the interval with endpoints 1 and x.

$[1, x]$, if $x > 1$, $[x, 1]$ if $x < 1$.

Domain of L is $\{x : x > 0\}$.

Where is L increasing?

- Use FTC II.
- 1st derivative for increasing functions

$$L(x) = \int_1^x \frac{1}{t}\,dt.$$

So $L'(x) = 1/x > 0$ if $x > 0$

L is increasing on ~~$[0, \infty)$~~. $(0, \infty)$.

Net change theorem.

Since F is an anti-derivative F', thus FTC I tells us

$$F(b) - F(a) = \int_a^b F'(t)\,dt.$$

– The integral gives us the <u>net</u> <u>change</u> on an ~~integral~~ interval.

Example Suppose that a particle moves so that its velocity $v(t) = \sin(t)$.

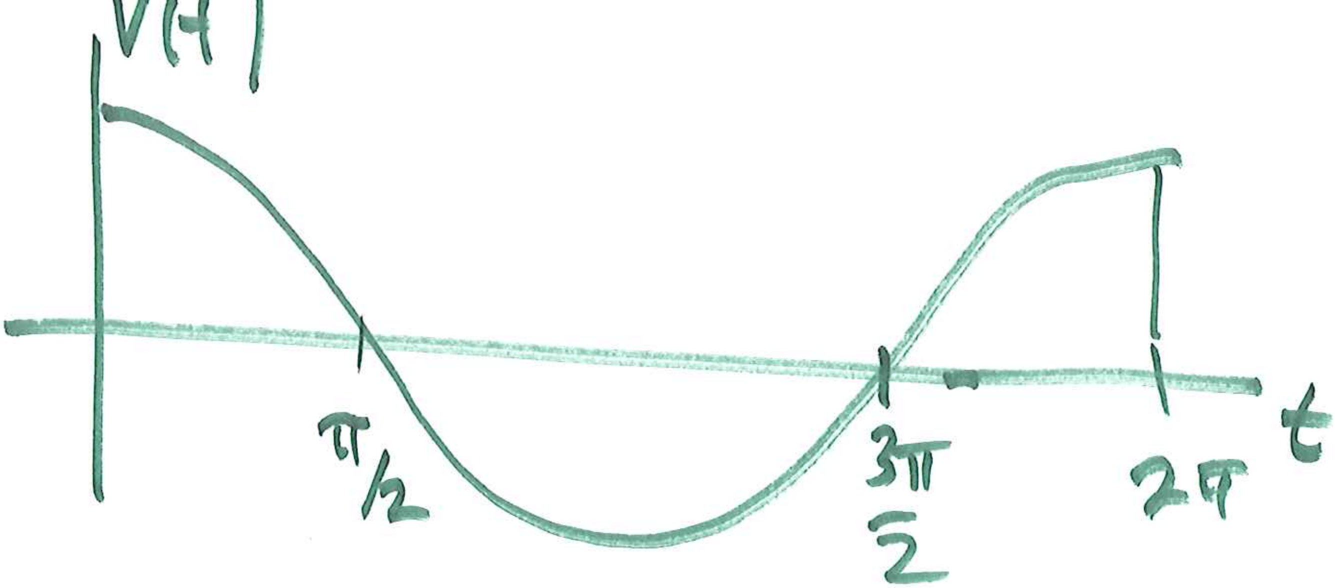
- Find the net. change of position for t in $[0, 2\pi]$
- Find the the total distance traveled for t in $[0, 2\pi]$

~~position is~~ velocity is the deriv. of position so if $p(t)$ is position. Net change is

$$p(2\pi) - p(0) = \int_0^{2\pi} v(t)\,dt$$

$$= \int_0^{2\pi} \sin(t)\,dt = 0.$$

Total distance.



$$\int_0^{\pi/2} v(t)\,dt - \int_{\pi/2}^{3\pi/2} v(t)\,dt + \int_{3\pi/2}^{2\pi} v(t)\,dt$$

$$= \int_0^{2\pi} |v(t)|\,dt.$$

Total dist.

$$\int_0^{\pi/2} \sin(t)\,dt = \cos(t)\Big|_0^{\pi/2}$$

$$= 1$$

$$-\int_{\pi/2}^{3\pi/2} \sin(t)\,dt = 2.$$

$$\int_{3\pi/2}^{2\pi} \sin(t)\,dt = 1$$

4 units.