Substitution rule $\underline{L39}$

Clickerless question

$$\int F'(g(x))\,g'(x)\,dx$$

$$= F(g(x)) + c$$

why? chain rule

<u>Example</u>

$$\int \overbrace{2x}^{g'(x)}\;\overbrace{\sin(x^2)}^{F'(g(x))}\,dx = -\cos(x^2) + c.$$

$F(x) = -\cos(x)$ $\quad g(x) = x^2$

$F'(x) = \sin(x)$ $\quad g'(x) = 2x$

Find

$$\int x e^{x^2} dx$$

$$= \frac{1}{2} \int \underbrace{2x}_{g'(x)} \overbrace{e^{x^2}}^{F(g(x))} dx$$

$$= \frac{1}{2} e^{x^2} + C.$$

Check. $\frac{d}{dx}\left(\frac{1}{2} e^{x^2} + C\right)$

$$= 2x \frac{1}{2} e^{x^2}. \checkmark$$

Substitution rule.

If F is an anti-derivative of f, then

$$\cancel{\int f(g(x))\,dx = F(g}$$

$$\int f(g(x))\,g'(x)\,dx = F(g(x)) + c.$$

Ritual of u-substitution.

Example Find

$$\int \frac{1}{(1-2x)^2}\,dx$$

<u>Example</u> Find

$$\int \frac{1}{(1-2x)^2}\,dx$$

Let $u = 1-2x = g(x)$.

$$\frac{du}{dx} = g'(x) = -2.$$

$$du = -2\,dx.$$

or $-\frac{1}{2}\,du = dx.$

$$\int \frac{1}{(\underbrace{1-2x}_{u})^2}\,\underbrace{dx}_{-\frac{1}{2}du} = -\frac{1}{2}\int \frac{1}{u^2}\,du$$

$$= -\frac{1}{2}\int u^{-2}\,du = -\frac{1}{2}\,\frac{u^{-1}}{-1} + c'$$

$$= +\frac{1}{2}\,\frac{1}{u} + c' = \underline{\frac{1}{2(1-2x)} + c'}$$

Example

$$\int \sin(x)\, dx$$

$$u = \sin(x)$$

$$du = \cos(x)\, dx \quad \text{or} \quad dx = \frac{du}{\cos(x)}.$$

$$\int \sin(x)\, dx = \int \frac{1}{\cos(x)}\, du.$$

$$= \,?$$

Need expression involving u.

Example

$$\int \sin(x)\cos(x)\,dx$$

Substitue $u = \sin(x)$

$$\frac{du}{\cancel{dx}} = \cos(x)\,dx$$

-or- $v = \cos(x)$

$$-dv = +\sin(x)\,dx.$$

$$\int \sin(x)\cos(x)\,dx = \int u\,du$$

$$= \frac{1}{2}u^2 = \frac{1}{2}\sin^2(x).$$

or.

$$\int \sin(x)\cos(x)\,dx = -\int v\,dv$$

$$= -\frac{1}{2}v^2 = -\frac{1}{2}\cos^2(x).$$

$$-\frac{1}{2}\cos^2(x) = \frac{1}{2}\sin^2(x) + d$$

$$\frac{1}{2}\left(\overset{1}{\cancel{\sin^2(x)+\cos^2(x)}}\right) = 0 + d.$$

$$\frac{1}{2} = 0. + d$$

Definite integrals & substitution

2 approaches.

1. ~~Use substiti~~

First consider ~~the~~ an indefinite integral & use substitution to find anti-derivative.

2. Evaluate definite integral using FTC I.

1. Change limits as we substitute

2nd approach.

$$\int_a^b f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du.$$

$u = g(x)$. $x=b$, $u=g(b)$

$x=a$, $u=g(a)$.

Find $\displaystyle\int_1^4 \sqrt{2x+1}\,dx$

Use $u = 2x+1$. $\quad x=1,\ u=3$

$\frac{1}{2}\,du = \frac{2dx}{2} \quad x=4,\ u=9.$

$$\int_1^4 \sqrt{2x+1}\,dx = \frac{1}{2}\int_3^9 \sqrt{u}\,du$$

$$= \frac{1}{2}\,\frac{u^{3/2}}{3/2}\Bigg|_{u=3}^{9}$$

$$= \frac{1}{3}\left(9^{3/2} - 3^{3/2}\right). = \frac{1}{3}\left(27 - 3\sqrt{3}\right).$$

$$= \underline{9 - \sqrt{3}}$$

Clicker question.

$$\int_1^3 \frac{1}{(1+2x)^2}\,dx = \int_3^7 \frac{1}{u^2}\,du \cdot \frac{1}{2}.$$

$u = 1 + 2x$

$x=1,\ u=3$ $\quad du = 2\,dx$

$x=3,\ u=7$ $\quad \frac{1}{2}\,du = dx$

$$\int x \sqrt{2x+1}\, dx$$

Try $u = 2x+1$, $du = 2dx$

$\frac{1}{2} du = dx.$

Solve $u = 2x+1$ for x

$$x = \frac{1}{2}(u-1).$$

$$= \int \frac{1}{2}(u-1)\sqrt{u}\ \frac{1}{2} du$$

$$= \frac{1}{4}\int u^{3/2} - u^{1/2}\, du.$$

Clicker

$$\int x\sqrt{2+3x}\,dx$$

$u = 2+3x$, $du = 3\,dx$.

$x = \frac{1}{3}(u-2)$ $\quad \frac{1}{3}du = dx$

$$= \frac{1}{3}\int \frac{1}{3}(u-2)\sqrt{u}\,du$$

$$= \frac{1}{9}\int u^{3/2} - 2\sqrt{u}\,du$$ A