12/4/15

1716. 202 pages

2011. 1368 pages

?. 2000 pages.

Assumption $N(t)$ = number of pages grows exponentially

$N(t) = Ae^{kt}$. Where t = # of years after 1716.

$N(t)$ = number of pages.

$$N(0) = A \cdot e^{0} = 202$$

$$N(295) = Ae^{k295} = 1368.$$

$A = 202.$

$$202e^{k295} = 1368$$

$$e^{k295} = 1368/202$$

$$k295 = \ln\left(\frac{1368}{202}\right).$$

$$k = \frac{1}{295}\ln\left(\frac{1368}{202}\right).$$

$$\approx 0.0064842$$

$$N(t) = 202e^{kt}.$$

Solve $N(t) = 2000$ for t.

$$2000 = 202e^{kt}.$$

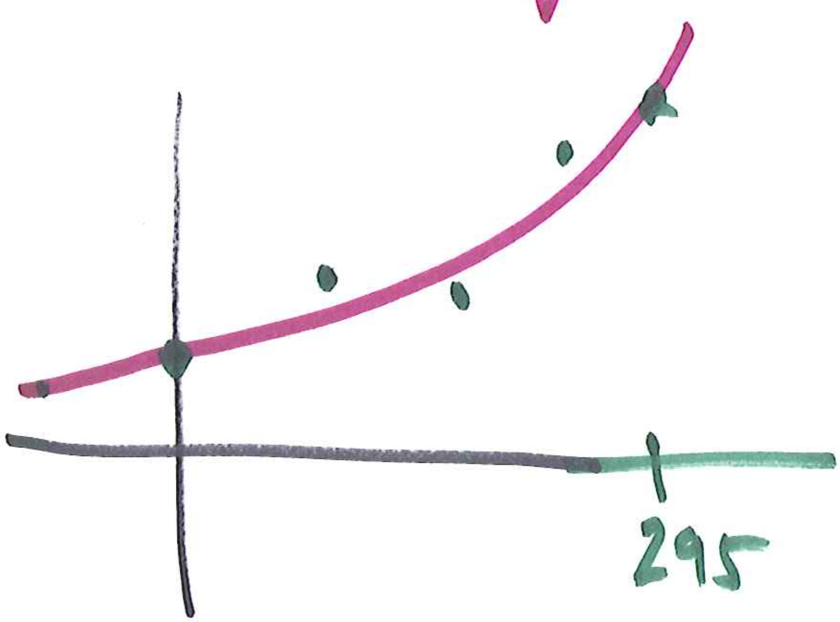
$$e^{kt} = 2000/202$$

$$\cancel{k}t = \frac{1}{k}\ln\left(\frac{2000}{202}\right)$$

$$\approx 354 \text{ years.}$$

after 1716

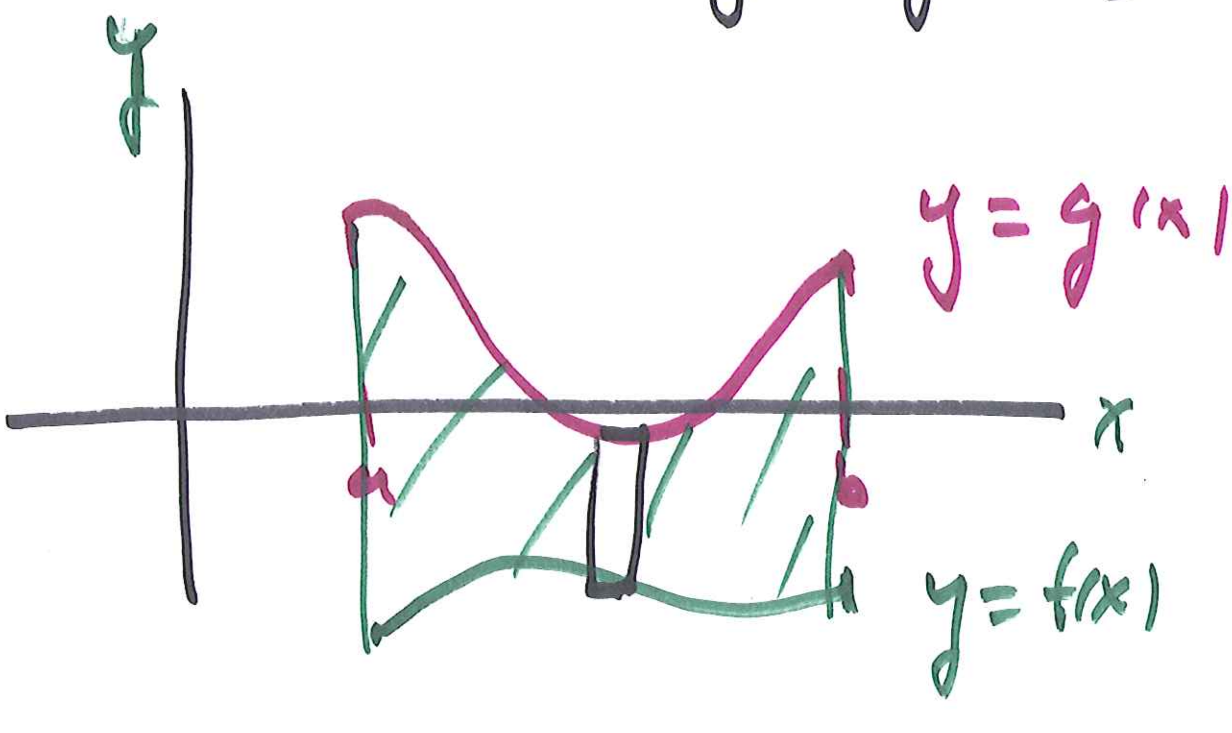
or in the year 2070.



Area.

- Find area of regions enclosed by several curves of the form $y = f(x)$.

Let $R = \{(x, y) : a \le x \le b, \; f(x) \le y \le g(x)\}$



Partition $[a, b]$ with

$a = x_0 < x_1 < \dots < x_n = b.$

For each interval $[x_{i-1}, x_i]$

Consider a thin rectangle

$$R_i = \{(x, y) : x_{i-1} \le x \le x_i$$

$$f(x_{i-1}) \le y \le g(x_{i-1}).\}$$

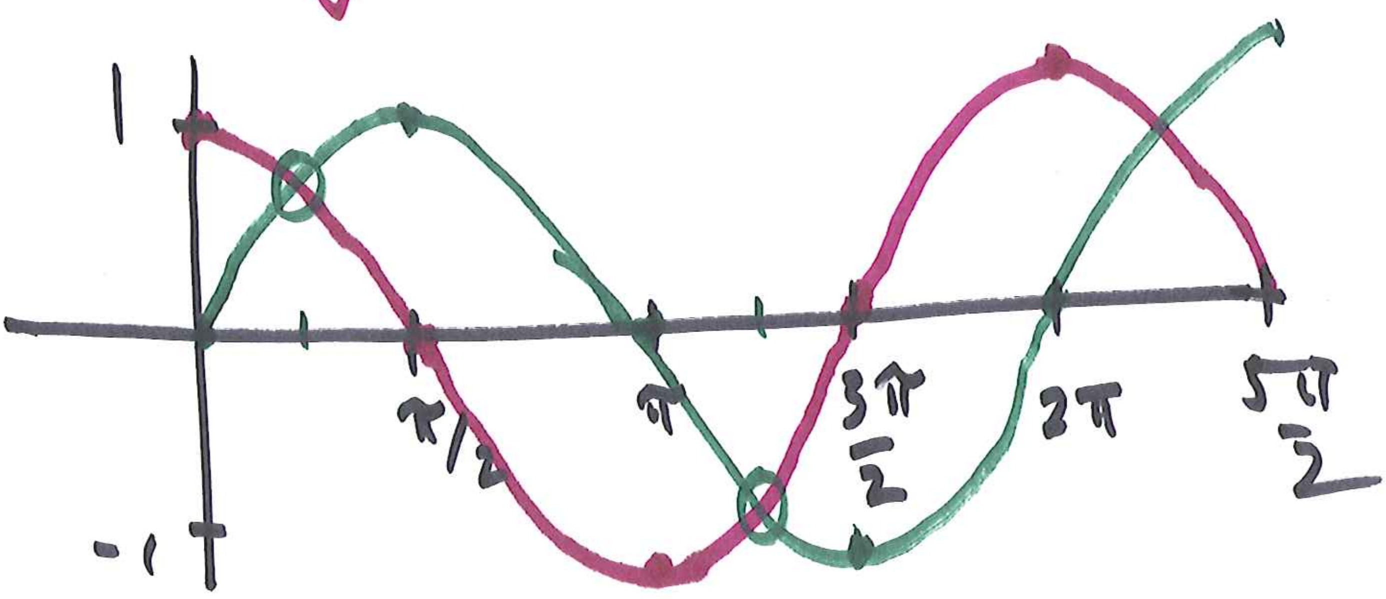
Area of R_i is $\cancel{\Delta x_i}$

$$(g(x_{i-1}) - f(x_{i-1}))\overbrace{(x_i - x_{i-1})}^{\Delta x_i}$$

Sum on i

$$\sum_{i=1}^{n} R_i = \sum_{i=1}^{n} (g(x_{i-1}) - f(x_{i-1}))\,\Delta x_i$$

<u>Area</u>: Find the area of one of the regions enclosed by $y = \sin(x)$ and $y = \cos(x)$.



Solve $\sin(x) = \cos(x)$.

Solutions are $\frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4},$

$\frac{\pi}{4} + k\pi, \quad k = 0, \pm 1, \pm 2, \ldots$

Take limit as largest subinterval goes to 0 and obtain

"Poof."

$$\text{area of } R = \int_a^b (g(x) - f(x))\,dx$$

Variations.

1. Finding a, b.
2. ~~Gen~~ x Consider curve the form $x = g(y)$

Find the area of

$$R = \left\{ (x,y) : \frac{\pi}{4} \le x \le \frac{5\pi}{4} \right.$$

$$\left. \cos(x) \le y \le \sin(x) \right\}$$

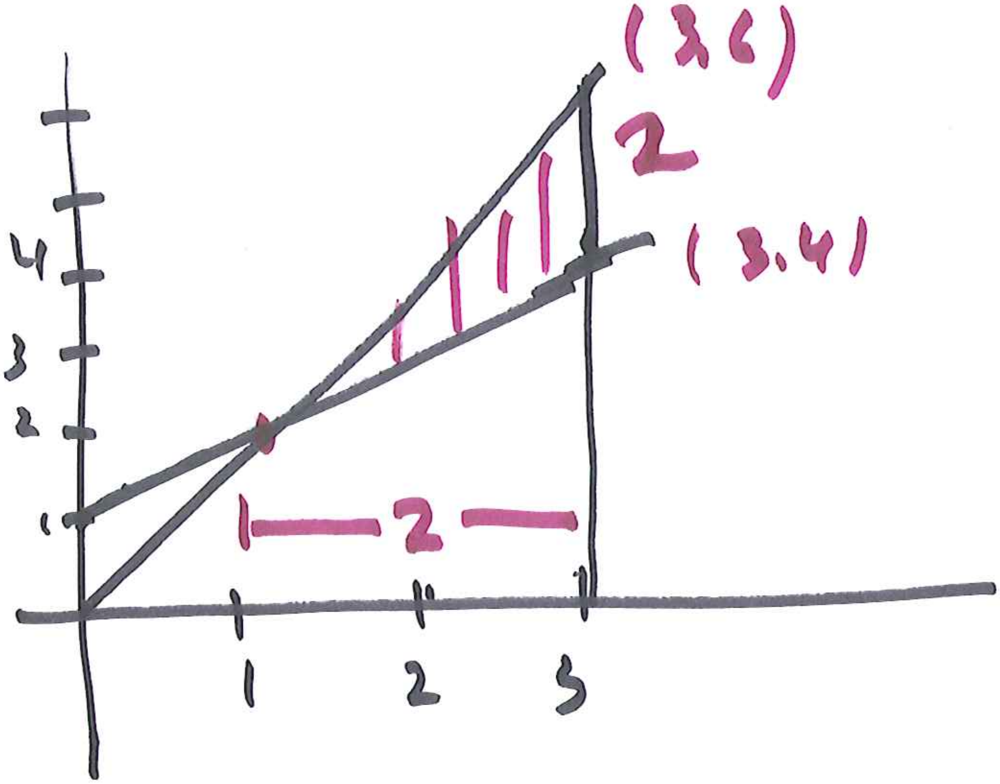
- Draw a picture!!!

$$\text{area of } R = \int_{\pi/4}^{5\pi/4} (\sin(x) - \cos(x))\,dx.$$

$$= -\cos(x) - \sin(x) \Big|_{x=\pi/4}^{5\pi/4}$$

$$= +\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$$

$$= \underline{2\sqrt{2}}.$$



$$\int_1^3 (2x-(x+1))dx$$

$$= \int_1^3 x-1\,dx$$

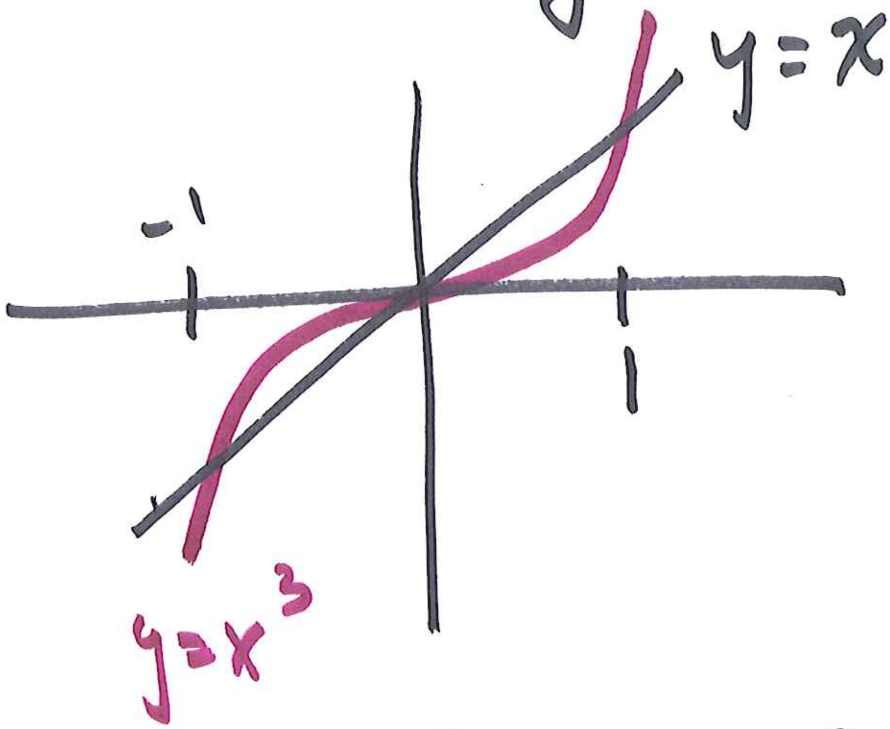
$$= \frac{x^2}{2} - x \Big|_1^3$$

$$= \frac{9}{2} - \frac{1}{2} - 3 + 1.$$

$$= 4 - 3 + 1 = 2$$

Find the area between

$y = x$ and $y = x^3$



Solve $x = x^3$ or $(x^3 - x) = 0$

$$x(x^2-1) = 0$$

$$x(x-1)(x+1) = 0$$

$$\text{Area} = \int_{-1}^{0} (x^3 - x)\,dx + \int_0^1 x - x^3\,dx$$

$$\vdots$$

$$= 1/2.$$

$$\int_{-1}^{1} x - x^3 \, dx = 0$$