

- Teaching evaluations!
- If you have 5 excused absences, please submit documentation.

$g(x) = \sqrt{x}$, find the inverse function.

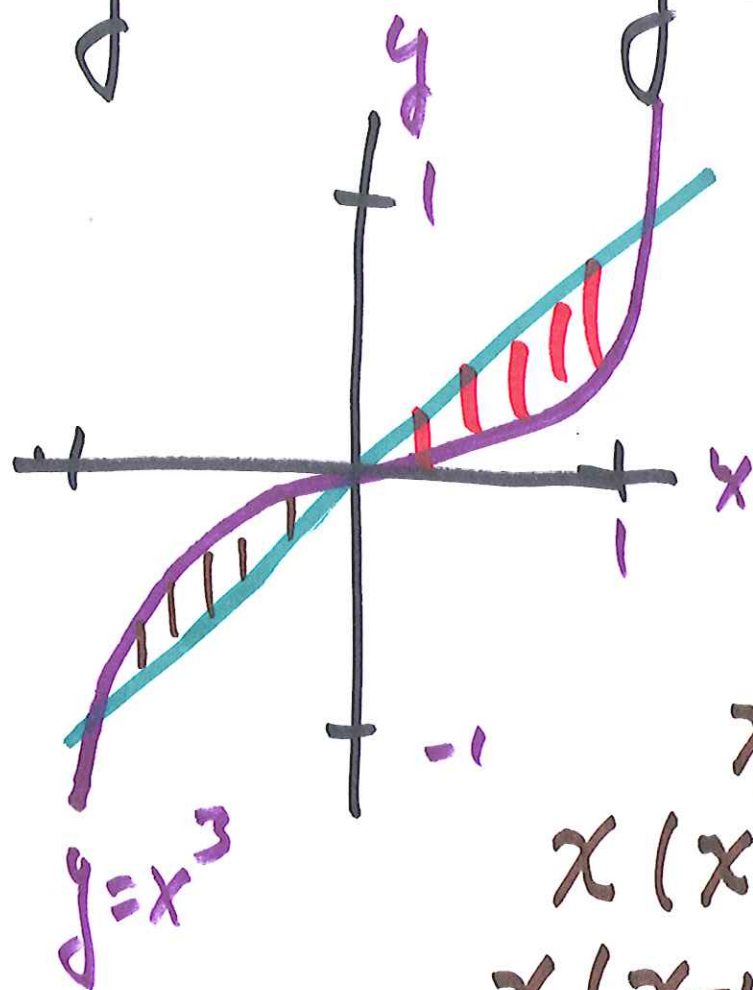
- Domain & range of g are $[0, \infty)$. Find $f = g^{-1}$

$$y = \sqrt{x}, \quad y^2 = x, \quad \text{~~find } f~~$$

$$x = f(y) = y^2 \quad \text{or} \quad \underline{\underline{f(x) = x^2}}$$

Find the area between

$$y = x \text{ and } y = x^3$$



$$y = x$$

Find points
of intersection.
Solve $x^3 = x$.

$$x^3 - x = 0$$

$$x(x^2 - 1) = 0$$

$$x(x-1)(x+1) = 0$$

Solutions are $x = 0$, $x = 1$, $x = -1$

Find area over $[-1, 0]$

Subtract upper function from
lower function and integrate.

$$\int_{-1}^0 x^3 - x \, dx$$

$$\int_0^1 x - x^3 dx$$

Total area is sum

$$\int_{-1}^0 x^3 - x dx + \int_0^1 x - x^3 dx$$

$$\left. \frac{x^4}{4} - \frac{x^2}{2} \right|_{-1}^0 + \left. \frac{x^2}{2} - \frac{x^4}{4} \right|_0^1$$

$$- \left(\frac{1}{4} - \frac{1}{2} \right) + \frac{1}{2} - \frac{1}{4}$$

$$= \underline{\underline{1/2}}$$

Area between f & g over interval $[a, b]$ is

$$\int_a^b |f(x) - g(x)| dx$$

Just as total distance
travelled for $t \in [a, b]$

$$\int_a^b |v(t)| dt$$

where $v(t)$ is velocity.

(1 point) local/rmb-problems/quad-inverse.pg

Let $f(x) = (x + 5)^2 + 5$. Find the largest value of a so that f is one to one on the interval $(-\infty, a]$.

 $a =$ -5

Consider f on the domain the interval $(-\infty, a]$ and let g be the inverse of the function f . Give the domain and the range of the function g .

The domain is the interval



and the range is

the interval



. help (intervals)

Give a formula for the function g . $g(x) =$ 

help (formulas)

Graph of f is a parabola w
vertex at $(-5, 5)$.



f is one-one on $(-\infty, -5]$

f on $(-\infty, -5]$. Domain.

has range $[5, \infty)$.

$g = f^{-1}$ has domain $[5, \infty)$

range $(-\infty, -5]$.

Find $g = f^{-1}$.

$$y = (x+5)^2 + 5.$$

$$(y-5) = (x+5)^2$$

$$\pm \sqrt{y-5} = (x+5).$$

$$x = -5 \pm \sqrt{y-5}$$

Choose neg. sign so that

$-5 - \sqrt{y-5}$ is in the range of f , $(-\infty, -5]$. So

$$g(y) = -5 - \sqrt{y-5}$$

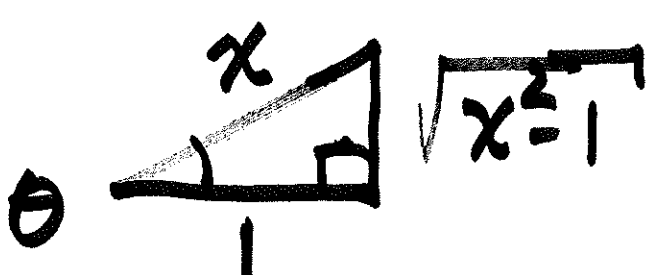
$$\text{or } g(x) = -5 - \sqrt{x-5}$$

Simplify by referring to the appropriate triangle or trigonometric identity.

(Use symbolic notation and fractions where needed.)

$$\cot(\sec^{-1}(x)) = \text{[input box]} \text{ help (fractions)}$$

Hint: Generate the appropriate triangle and use the definition of a trigonometric function and its inverse.

$$\theta = \sec^{-1}(x), \quad \sec(x) = \frac{1}{\cos(x)} = \frac{\text{hyp}}{\text{adj.}}$$


$$\begin{aligned} \cot(\sec^{-1}(x)) &= \cot(\theta) \\ &= \frac{1}{\tan(\theta)} = \frac{\text{adj}}{\text{opp}} \\ &= \frac{1}{\sqrt{x^2 - 1}} \end{aligned}$$

(1 point) local/rmb-problems/exp-alg2.pg

A function f is given by the formula $f(x) = A \cdot e^{kx}$ for constants A and k . We also know that $f(2) = 8$ and $f(3) = 1$. Find numerical values for the constants A and k .

$A =$ , $k =$  help (numbers).

The function f is ? .

$$\begin{array}{l|l}
 f(2) = A e^{2k} = 8 & e^k = \frac{1}{8} \\
 f(3) = A e^{3k} = 1 & \ln(e^k) = \ln\left(\frac{1}{8}\right) \\
 \frac{f(3)}{f(2)} = \frac{A e^{3k}}{A e^{2k}} = \frac{1}{8} & k = -\ln(8)
 \end{array}$$

Find A .

$$\begin{array}{l|l}
 A e^{-2\ln(8)} = 8 & A = 8^{+3} \\
 A 8^{-2} = 8 & f(x) = 8^3 e^{-x\ln(8)} \\
 & = 8^3 \cdot 8^{-x}
 \end{array}$$

A stone is tossed in the air from ground level with an initial velocity of 30 m/s. Its height at time t is $h(t) = 30t - 4.9t^2$ m.

Compute the stone's average velocity over the time interval $[1, 3.5]$.

Average velocity =



$$\begin{aligned} &\text{Average velocity on } [1, 3.5] \\ &\text{is } \frac{h(3.5) - h(1)}{3.5 - 1} \frac{\text{m}}{\text{s}} \\ &= \frac{19.875}{2.5} \text{ m/s} \\ &= 7.95 \text{ m/s.} \end{aligned}$$

Evaluate the limit assuming that $\lim_{x \rightarrow 2} g(x) = 10$:

$$\lim_{x \rightarrow 2} \frac{g(x)}{x^2} = \boxed{} \quad \boxed{\begin{smallmatrix} \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \end{smallmatrix}}$$

Quotient rule for computing limits

$$\lim_{x \rightarrow 2} \left(\frac{g(x)}{x^2} \right) = \frac{\lim_{x \rightarrow 2} g(x)}{\lim_{x \rightarrow 2} x^2}$$

Provided

$$\lim_{x \rightarrow 2} x^2 \neq 0$$

$$\begin{array}{c} 11 \\ 4 \end{array}$$

$$= \frac{10}{4} = \frac{5}{2}$$