L 43 - Review:

A function f is continuous at 0 if

$$f(0) = \lim_{x \to 0} f(x).$$

If $f(x) = 42x + \sin(x)\csc(x)$.

then $\sin(x) \cdot \csc(x)$

$$= \sin(x) \frac{1}{\sin(x)} = 1.$$

So $f(x) = 42x + 1$

$$f'(x) = 42.$$

A2.4 #14

Let f be defined by

$$f(x) = \begin{cases} x+5, & x \le 2 \\ Ax+B, & 2 < x < 7 \\ 2x+13 & 7 \le x \end{cases}$$

Find the values of A and B which make f continuous everywhere.

$A =$ $B =$

(numbers)

Need

$$f(2) = \lim_{x \to 2^-} (x+5) = \lim_{x \to 2^+} Ax+B$$

$$7 = 7 = \underline{2A+B}$$

$$f(7) = 27 = \lim_{x \to 7^-} Ax+B = \lim_{x \to 7^+} 2x+13$$

$$= 27 = \underline{7A+B = 27}.$$

Solve $-(2A + B = 7)$

$$7A + B = 27$$

$$5A + 0 = 20$$

$$\underline{A = 4}$$

$$\underline{B = -1}$$

A2.5 #7

i) Decide if the following limit exists $\lim_{x \to \frac{-\pi}{4}} \frac{\sin x + 1 \cdot \cos x}{\tan x + 1}$:

- Yes
- No

ii) If so, evaluate it. Otherwise enter DNE.

(Use symbolic notation and fractions where needed.)

limit =

– Algebraic or L'Hôpital.

$$\frac{(\sin(x) + \cos(x))\cos(x)}{\left(\frac{\sin(x)}{\cos(x)} + 1\right)\cos(x)}$$

$$= \frac{\cancel{\sin(x) + \cos(x)}}{\cancel{\sin(x) + \cos(x)}} \cdot \cos(x).$$

$$\lim_{x \to -\pi/4} \cos(x) = \frac{\sqrt{2}}{2}$$

A2.8 #5

Find an interval (a, b) of length 1 and so that we may use the values of the expression $xe^{\frac{x}{2}}$ at the endpoints of the interval and the intermediate value theorem to show that the equation

$$xe^{\frac{x}{2}} = 2$$

has a solution in the interval (a, b).

$a =$ ______ $b =$ ______

$f(x) = xe^{x/2}$ is continuous

$f(0) = 0$

$f(1) = e^{1/2} \approx 1.65 < 2$

$f(2) = 2 \cdot e \approx 5.4 > 2$

There is a solution on the interval $[1, 2]$.

B3.1 #6

Find $f(3)$ and $f'(3)$, assuming that the tangent line to $y = f(x)$ at $x = 3$ has equation $y = 6x + 2$.

$f(3) =$

$f'(3) =$

Tangent line at $x = 3$
has equation

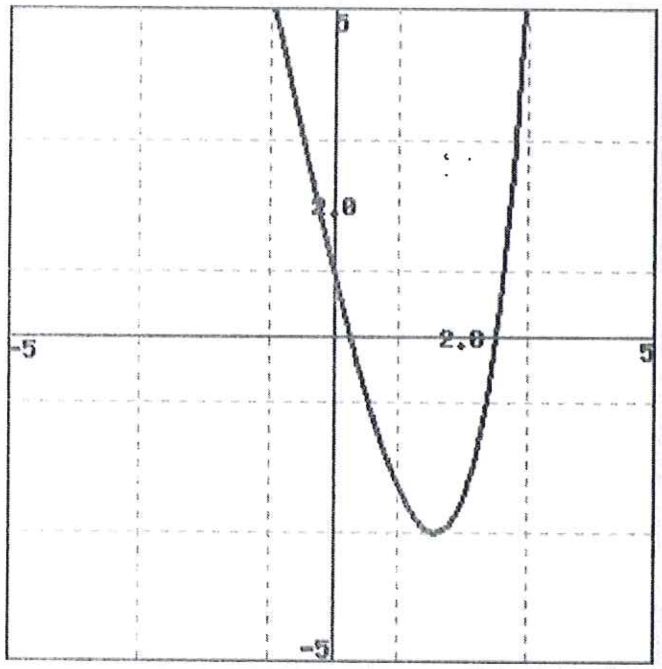
$$y - f(3) = f'(3)(x-3).$$

If $x = 3$ $\quad y = f(3) = 6 \cdot 3 + 2$
$= \underline{20}$

$f'(3)$ is slope of $y = 6x + 2$
or $\underline{6}$

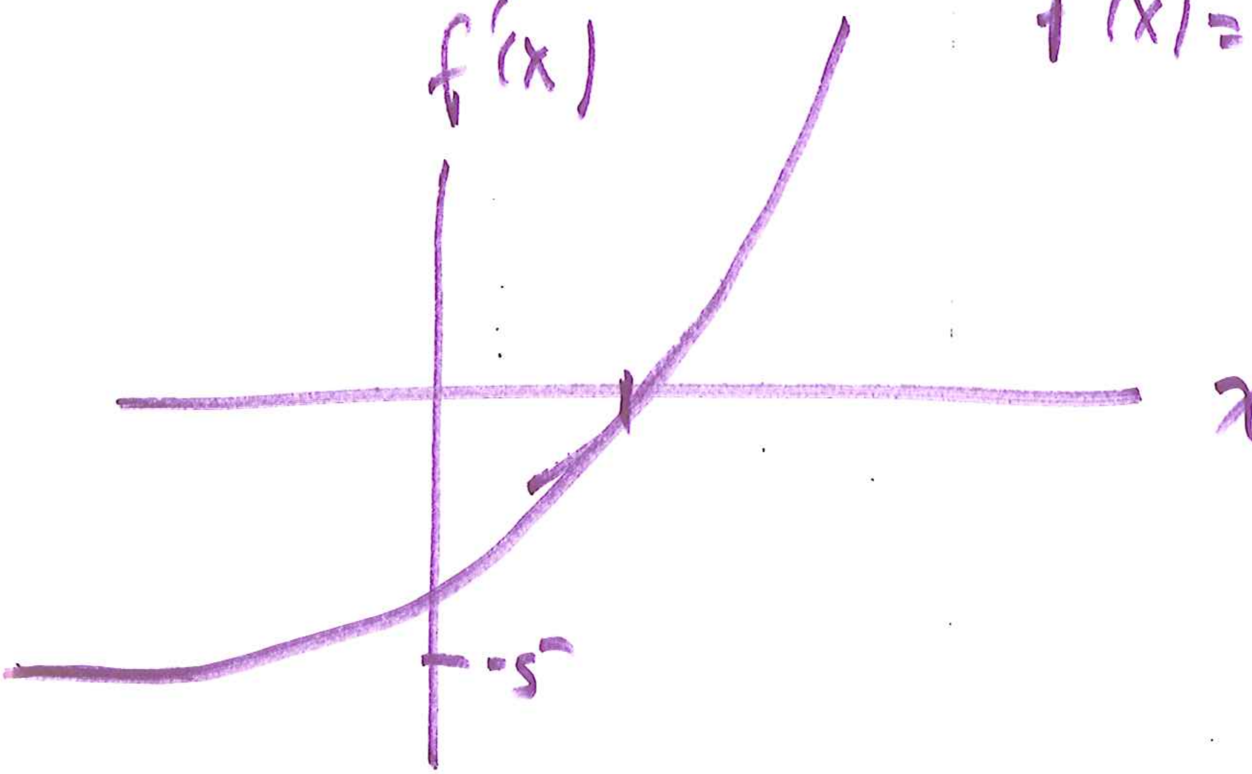
B3.2 #13

The following is a graph of the function $f(x) = e^x - 5x$:



Sketch the graph of $f'(x)$.

$f'(x) = e^x - 5$



B3.6 #4

If $f(x) = 10\sin^2(x)$ then $f'(x) =$

$f(x) = 10\sin^2(x)$

$\frac{d}{dx}(10\sin^2 x) = 10\frac{d}{dx}\sin^2(x)$.

$= 10 \cdot 2\,(\sin(x))^{2-1} \cdot \frac{d}{dx}\sin(x)$

$= \underline{10 \cdot 2\sin(x)\cos(x)}$.

$= 20\sin(x)\cos(x)$.

Clicker-mania

$$\frac{d}{dx}\left(x\ln(2x)\right)$$

$$= \left(\frac{d}{dx}x\right)\ln(2x) + x\,\frac{1}{2x}\,\frac{d}{dx}(2x)$$

$$= 1\cdot\ln(2x) + \frac{1}{2}\cdot 2$$

$$= \ln(2x) + 1.$$

— or —

$$= \frac{d}{dx}\left(x\left(\ln(x)+\ln(2)\right)\right)$$

$$= 1\cdot\left(\ln(x)+\ln(2)\right) + x\cdot\frac{1}{x}$$

$$= \ln(2x) + 1.$$

B3.7 #9

Given the following functions: $f(x) = \cos(x)$ and $g(x) = x^7 + 1$. Find:

$\frac{d}{dx} f(g(x)) =$

$\frac{d}{dx} g(f(x)) =$

$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$

$-\sin(x^7+1) \cdot 7 \cdot x^6$

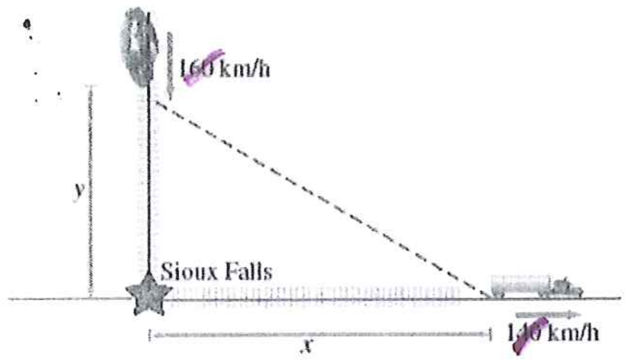
$\frac{d}{dx} g(f(x)) = g'(f(x))\, f'(x)$

$= 7 \cdot \cos^6 x \cdot (-\sin(x)).$

$= -7\cos^6(x) \cdot \sin(x)$

B3.11 #10

A police car is traveling south on Hwy 43 toward Sioux City at 180 km/h and a truck is traveling east away from Sioux City, IA, at 120 km/h (See figure below).



At time $t_0 = 0$, the police car is 50 km north and the truck is 30 km east of Sioux Falls.

Calculate the rate at which the distance between the vehicles is changing after 10 minutes.

(Use decimal notation. Give your answer to three decimal places.)

The rate of change of the distance between the vehicles is

$t = 10$ min $\frac{1}{6}$ hr.

Car is $50 - \frac{1}{6}180 = 50 - 30 = \cancel{20\text{ km}}$
$= 20$ km
north of Sioux Falls.

Truck is $30 + \frac{1}{6}120 = 50$ km

Consider

$$f(t) = \sqrt{x^2 + y^2}$$

$$f'(t) = \frac{1}{2}(x^2+y^2)^{-1/2}(2xx' + 2yy')$$

$$= \frac{xx' + yy'}{\sqrt{x^2+y^2}}$$

$x = 50 \qquad y = 20$

$x' = 120 \qquad y' = -180$

$$f'(t) = \frac{50 \cdot 120 - 20 \cdot 180}{\sqrt{50^2 + 20^2}} \ \frac{\text{km}}{\text{hr}}$$

at $t = 1/6$.

C4.7 #2

Find a positive number x such that the sum of $25x$ and $\frac{1}{x}$ is as small as possible.

$x =$

Does this problem require optimization over an open interval or a closed interval?

~~A. open~~

B. closed

$f(x) = 25x + \frac{1}{x}$. Find smallest value for f on $(0, \infty)$.

Compute $f'(x) = 25 - \frac{1}{x^2}$

$f'(x) > 0$ for $25 - \frac{1}{x^2} > 0$ or $x^2 > 1/25$ or $x > 1/5$.

$f'(x) < 0$ for $0 < x < 1/5$.

Since f is increasing on $(\frac{1}{5}, \infty)$ & decreasing on $(0, 1/5)$, $f(1/5)$ is ~~a minimum~~ the smallest value.